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# ILLUSION OR REALITY? THE MATHEMATICS OF ATTAINABLE GOALS AND IRREDUCIBLE UNCERTAINTIES†

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In complex, high-dimensional systems, it is usually far from obvious what states are attainable within the constraints on admissible actions. Similarly, when only parts of the system are physically measurable, a vital practical as well as philosophical question arises as to how much inherent uncertainty remains in determining the true state of the system. In system-theoretic jargon, these are problems of *reachability* and *constructibility*.

This paper presents an overview of the current mathematical state-of-the-art as it relates to the reachability/constructibility question. Particular emphasis is given to those results which seem most useful for dealing with practical system problems. After an introductory section to motivate the subject, a survey of the principal mathematical results for linear systems is given, along with a discussion of multidimensional nonlinear problems. The paper concludes with several actual problems from ecology, urban systems, water resource systems, and transportation networks where reachability and/or constructibility questions play an important role in the analysis.

INDEX TERMS System theory, controllability, reachability, observability.

## INTRODUCTION

onsider the plight of a Government Economic dvisor *A*, who is concerned with management f the economy. At his disposal are a set of ctions (controls) such as regulation of interest ates, imposition of tariffs, change of money upply and the like. The economy is in a certain ate, at present, and the technocrat would like to ransfer it to a different, presumably more atisfactory state within a given period of time before the next election), perhaps subject to onstraints on unemployment levels, rate of nflation, etc. Economist *A*'s first question might e: are the tools at my disposal sufficient to ccomplish the task or, in other words, is there *ny* combination of actions which will achieve he desired result?

Now consider the equally vexing problem ced by his colleague down the hall, Economist . Various indicators purporting to reflect the urrent state of the economy are computed. mong these are such indices as the nemployment rate, the consumer price index, gross national product, and so forth. On the basis of these measured indicators, Economist *B* would like to be able to determine the "true" state of the economy so that he could report it to *A* in the hopes that appropriate and timely actions may be taken. The basic question faced by Economist *B* is whether the measured information is sufficient to construct the true state of the economy or if there are parts of the internal economic mechanism that are not reflected in the measured output.

The problem faced by Economist *A* is a problem of *reachability*: with a given set of inputs, can a given dynamical process be driven from its current state to a prescribed state within a certain period of time?

Economist *B* is facing a question of *constructibility*: on the basis of system outputs measured over a certain length of time, can the internal state of a given dynamical process be determined exactly?

Although the foregoing questions are part of the much broader underlying question: *how can mathematics contribute to systems analysis*?, in this report we shall merely try to outline what has happened and might be expected to happen within the narrow confines of determining attain-

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able goals and ascertaining the limitations of information from measurement as these problems pertain to modern systems.

A moment's thought is sufficient to be convinced that the properties of reachability and constructibility are fundamental system concepts in that they delineate what is and is not possible within given structure and interaction constraints. If one accepts the view that good systems analysis, like good politics, is the art of the possible, then it is difficult to escape the conclusion that a thorough investigation of the reachable and constructible states should be carried out at a very early stage in the analysis of any model proposed to represent a given system. There is no utility in striving to achieve *theoretically* unobtainable goals or in attempting to resolve *fundamentally* indistinguishable situations.

It is our intention in this paper to briefly survey the current state of the art in the mathematical system theory world as it pertains to the reachability/constructibility issue. We shall strive to present the results in as intuitive a manner as possible, preferring in many places to replace absolutely precise statements of various results by weakened, but more transparent, versions conveying the essence of the original result. In this connection, we shall omit all proofs of theorems concentrating our attention instead upon the meaning and relevance of the results to problems of applied systems analysis.

## II SURVEY OF METHODOLOGY

In a brief paper such as this, it is clearly impossible to do justice to the vast literature on reachability/constructibility and to present even a small fraction of the useful and important results. Thus, what follows represents a somewhat eclectic selection of basic results which are either particularly simple, useful, and/or indispensable for dealing with the basic issues involved.

For the purpose of basic definitions, we consider the system $\Sigma$ described by the equations

$$\Sigma: \quad \dot{x} = f(x, u, t), \quad x(t_0) = x^0 \tag{1}$$

$$y(t) = h(x, t) \tag{2}$$

For simplicity in this paper, we assume that $f($ $0, t) = 0$ $h(0, t) = 0$, for all $t$ and that $f$ and $h$ ar analytic functions of their arguments with $x \in R$ $u \in R^m$. We let $\phi(t; \tau, x, u)$ denote the solution (1) corresponding to an initial state $x$ at time under application of the input function $u(s)$, $s \leqq t$.

DEFINITION 1 An event $(\tau, x^*)$ is *controllable* and only if there exists a $\bar{t} > t_0$, and an admissibl input $u \in \Omega$, (both $\bar{t}$ and $u$ may depend on $(\tau, x^*$ such that

$$\phi(\bar{t}; \tau, x^*, u) = 0$$

$\Sigma$ is *completely controllable* if it is controllable fo every event $(\tau, x^*)$.

In other words, an event is controllable if an only if it can be transferred to the origin in finit time by application of some admissible inpu function $u$.

DEFINITION 2 An event $(\tau, x^*)$ is *reachable* and only if there is an $\bar{s} > 0$, and a $u \in \Omega$ (both and $u$ may depend upon $(\tau, x^*)$), such that

$$x^* = \phi(\tau; \bar{s}, 0, u).$$

$\Sigma$ is *completely reachable* if and only if it reachable for every event $(\tau, x^*)$.

Graphically, controllability and reachabilit are as in Figure 1.

It is important to note that controllability an reachability are entirely different concepts. The coincide only in special cases, one of which when $\Sigma$ is a constant, continuous time, linea system. *Caution*! Even if a linear system completely reachable and completely controllabl it does not follow that any event $(\tau, x)$ may b transferred to any other event $(\tau_1, x_1)$, $\tau_1 \geqq \tau$, b suitable choice of input.

The notions dual to controllability/reachabilit are those of constructibility and observability Basic definitions are:

DEFINITION 3 Two events $(\tau, x_1)$, $(\tau, x_2)$ of dynamical system belong to the same observatio class (are indistinguishable in the future) if an only if

$$h(\phi(t; \tau, x_1, u), t) = h(\phi(t; \tau, x_2, u), t),$$

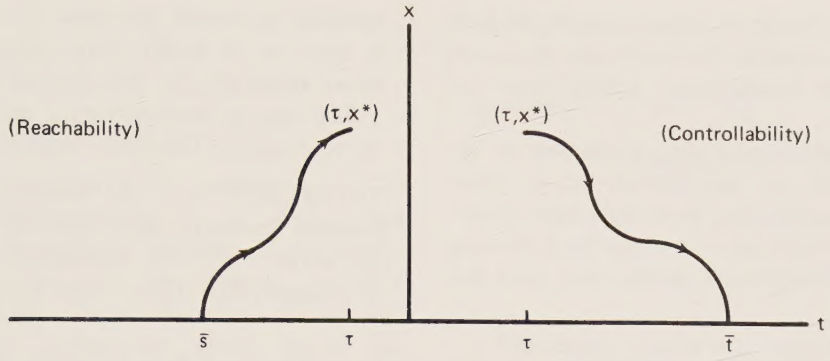


FIGURE 1

r all $t \geqq \tau$ and all inputs $u(t)$, $t \geqq \tau$.

The complementary notion is given by

EFINITION 4 Two events $(\tau, x_1)$, $(\tau, x_2)$ of a ynamical system belong to the same econstruction class (are indistinguishable in the ast) if and only if

$$h(\phi(\sigma;\tau,x_1,u),\sigma) = h(\phi(\sigma;\tau,x_2,u),\sigma)$$

r all $\sigma \leqq \tau$ and all $u(t)$, $\tau \leqq t \leqq \sigma$.

These definitions are needlessly elaborate in the near case. It is easily verified that if $f$ and $h$ are near, we can re-phrase Definitions 3 and 4 as

EFINITION 3′ An event $(\tau, x)$ of a linear ynamical system is *unobservable* if and only if it elongs to the observation class of $(\tau, 0)$, i.e., if nd only if

$$h(\phi(t;\tau,x,0),t) = 0$$

r all $t \geqq \tau$.

EFINITION 4′ An event $(\tau, x)$ of a linear ynamical system is *unconstructible* if and only if belongs to the reconstruction class of $(\tau, 0)$, i.e., and only if

$$h(\phi(\sigma;\tau,x,0),\sigma) = 0$$

r all $\sigma \leqq \tau$.

The motivation for Definition 3′ is clear: the ccurrence" of an unobservable event cannot be etected by looking at the output of the system ter time $\tau$. Definition 4′ says that the current ate of a system at time $\tau$ cannot be determined by its past output up to time $\tau$ if that state is unconstructible.

A. *Linear Systems*

We consider the constant, linear version of $\Sigma$ in which

$$\begin{aligned} f(x,u,t) &= Fx + Gu, \quad x(0) = 0 \\ h(x,t) &= Hx \end{aligned}$$

where $F$, $G$, $H$ are constant $n \times n$, $n \times m$, $p \times n$ matrices, respectively. In this case, the notions of reachability and controllability coincide, as do observability and constructibility. Due to the pioneering efforts of Kalman and others, we have at our disposal very detailed descriptions of the sets of reachable and constructible states. The single most important result in this direction is the following re-statement of a result first presented in Ref. 3.

THEOREM 1 *Define the $n \times nm$ controllability matrix $\mathscr{C}$ as*

$$\mathscr{C} = [G \mid FG \mid \cdots \mid F^{n-1}G].$$

*Let $\Omega$ be the set of bounded measurable functions on $[0, \infty]$. Further, let $\mathscr{L} = \{c^{(1)}, c^{(2)}, \ldots, c^{(r)}\}$ be a maximal linearly independent set of vectors from the columns of $\mathscr{C}$. Then the set $\mathscr{K}$ of reachable (or controllable) states of $\Sigma$ coincides with the subspace of $R^n$ spanned by the set $\mathscr{L}$.*

*Remarks:*

1) The fact that the reachable set is a subspace insures that it is convex.

2) In general, the set $\mathscr{L}$ is not unique. All that is needed to characterize the reachable set is *any* maximal linearly independent subset from the columns of $\mathscr{C}$.

3) Given a particular $(\tau, x^*)$ which lies in $\mathscr{L}$, Theorem 1 gives no information as to what control $u(t) \in \Omega$ would lead from the origin to $x^*$. In general, one would have to solve the following Fredhom integral equation of the first kind for such an input

$$x^*(\tau) = \int_0^\tau \exp(F(\tau - s))Gu(s)ds.$$

An alternate prescription involving generalized inverses is described in Ref. 15.

4) Theorem 1 remains partially valid if the continuous system (4) is replaced by the discrete-time system

$$x_{k+1} = Fx_k + Gu_k.$$

The reachable set is still given by the set $\mathscr{L}$, however, the controllable set is given by $\mathscr{L}$ only if the additional condition $\det F \neq 0$ is imposed.

Some useful corollaries of Theorem 1 are

COROLLARY 1 *A constant system $\Sigma = (F, G, -)$ is completely reachable if and only if there is no nontrivial characteristic vector of $F$ which is orthogonal to every column of $G$.*

COROLLARY 2 *A constant system $\Sigma = (F, G, -)$ is completely reachable if and only if the smallest $F$-invariant subspace of $R^n$ containing the columns of $G$ is $R^n$ itself.*

COROLLARY 3 *The state space of $\Sigma$ may be decomposed into the direct sum*

$$R^n = X_1 \oplus X_2,$$

*which decomposes the system dynamics as*

$$\frac{dx_1}{dt} = F_{11}x_1 + F_{12}x_2 + G_1u(t),$$

$$\frac{dx_2}{dt} = F_{22}x_2.$$

*The subsystem $(F_{11}, G_1, -)$ is completely reachable.*

*Remark* In a problem in which control enters, only the space $X_1$ has meaning. Thus, it is essential to isolate the space $X_1$ at the outset it may be of much lower dimension than t entire space $R^n$.

The case of time-varying $F$ and $G$ is somewh more complex. The basic result is

THEOREM 2[2] *Let the matrices $F(t)$, $G(t)$ bounded on every finite interval $t_0 \leqq t < \infty$. Th an event $(\tau, x^*)$ is reachable if and only $x^* \in$ range $[\hat{W}(s, \tau)]$, for some $s < \tau$, where*

$$\hat{W}(s, \tau) = \int_s^\tau \Phi_F(\tau, \sigma)G(\sigma)G'(\sigma)\Phi_F'(\tau, \sigma)d\sigma,$$

*with $\Phi_F(\tau, s)$ being the transition matrix of $F(t$ i.e.*

$$\frac{\partial}{\partial t}\Phi_F(t, s) = F(t)\Phi_F(t, s), \quad \Phi_F(s, s) = I.$$

*Remarks*

1) The reachable set $\mathscr{R}$ now depends upon and we have

$$\mathscr{R}(\tau) = \text{range } \hat{W}(t_1, \tau),$$

where $t_1$ is any value of $t$ for which $\hat{W}(t, \tau)$ h maximal rank.

2) If $G(\cdot)$ is zero on $(-\infty, \tau)$, we cannot ha reachability of $(\tau, x^*)$.

3) Making the substitution $\tau \to t$, $s \to 2\tau - t$, define a new matrix $W(\tau, t)$ which gives t analogue of Theorem 2 for controllability.

Since the rank condition implied by rema (1) may not be easy to verify in practice, we gi a simpler condition for "analytic" systems; i. those systems for which $F(t)$, $G(t)$ are (rea analytic functions of $t$.

THEOREM 3[19] *Let $F(t)$, $G(t)$ be (real) analyt functions on $t_0 \leqq t \leqq \infty$. Define the sequence matrix functions $Q_i(t)$ by*

$$Q_0(t) = G(t),$$

$$Q_{i+1}(t) = F(t)Q_i(t) - \dot{Q}_i(t),$$

$$i = 0, 1, \ldots, n-1.$$

*Then the linear system $\Sigma = (F(t), G(t), -)$ completely reachable at time $\tau$ if and only if t*

*rank of matrix*

$$\mathscr{C}(t)=[Q_0(t)\,|\,Q_1(t)|\cdots|Q_{n-1}(t)]=n$$

*for some time* $t\leqq\tau$.

Before considering questions of observability/constructibility, it is useful and instructive to examine a number of frequently occurring variants of the basic problem considered above. These variants all involve imposing some restrictions either upon the desired region of state space or upon the region of admissible controls.

We begin by weakening the notion of controllability. In many cases, rather than transferring an initial state $x^0$ to the origin we are concerned with transferring it to some subspace $\mathscr{K}\subset R^n$. For example, $\mathscr{K}$ may represent some set of equally desirable states, a set of terminal states, etc. More precisely, we have

DEFINITION 5 A system is called *reachable relative to the subspace* $\mathscr{K}=\{x:Kx=0\}$ if for every state $x^0\in R^n$, there exists a number $\bar{t}<\infty$, $0\leqq t\leqq\bar{t}$, such that $Kx(t)=0$.

The basic result, essentially a corollary of Theorem 1, is

THEOREM 4[5] *The system* $\Sigma=(F,G,-)$ *is reachable relative to the subspace* $K$ *if and only if*

$$\operatorname{rank}[KG\,|\,KFG|\cdots|KF^{n-1}G]=\operatorname{rank}K,$$

i.e.,

$$\operatorname{rank}K\mathscr{C}=\operatorname{rank}K.$$

We have already seen that the set of controllable states forms a subspace which is generated by the columns of $\mathscr{C}$. However, in some cases we are given a particular subspace $\mathscr{M}\subset R^n$ and would like to determine whether or not every state in $\mathscr{M}$ is controllable. This situation is of particular importance in those processes for which we have either *a priori* knowledge of or influence over the initial state $x^0$ and can assert that it belongs to a particular subspace of $R^n$.

The theorem settling the above question is

THEOREM 5 *The system* $\Sigma=(F,G,-)$ *is conditionally controllable from a subspace* $\mathscr{M}=\{x^0=My:y\in R^n\}$ *if and only if*

$$\operatorname{rank}[M\,|\,FG|\cdots|F^{n-1}G]=\operatorname{rank}[G\,|\,FG|\cdots|F^{n-1}G],$$

i.e.,

$$\operatorname{rank}[M\,|\,\mathscr{C}]=\operatorname{rank}\mathscr{C}.$$

The reader should note carefully the distinction between *relative* and *conditional* controllability. In the first instance we are concerned with whether every initial state in $R^n$ can be transferred to a given subspace, while the second concept involves the question of whether those states in a given subspace may be transferred to the origin.

Relative and conditional controllability have dealt with restrictions on the state space $R^n$ and their effect upon the controllability properties of the system. Now we consider restrictions upon the controls $u(t)$. As one might expect, imposition of constraints on the admissible inputs may have far-reaching consequences for the controllability/reachability of a system.

We define the input space

$$\Omega^+=\{\mathrm{u}(\mathrm{t}):u(t)\text{ is piecewise continuous and }u_i(t)\geqq 0,\text{ for all }t\},\quad i=1,\ldots,m.$$

Then the problem of *positive controllability* is to determine whether the system $\Sigma$ is controllable/reachable with inputs from $\Omega^+$. It is clear that if $\Sigma$ is positively controllable, then necessarily a controllable state $x_0$ has the form

$$x_0=\sum_{k=1}^{m}\sum_{i=0}^{\infty}\mu_{ik}F^iG^{(k)},\quad(-1)^{i+1}\mu_{ik}>0,$$
$$k=1,\ldots,m,\quad i=0,1,\ldots,$$

where $G^{(k)}$ is the $k$th column of $G$. Obviously, each positively controllable system is controllable but not conversely.

Various conditions have been developed to check for positive controllability since the initial paper in the area.[27] For single-input systems, the most interesting is given by the following result:

THEOREM 6[4] *Let* $[a(t)]_-$ *denote the negative part of the function* $a(t)$, *i.e.*,

$$[a(t)]_- = \begin{cases} a(t), & a(t) < 0, \\ 0, & a(t) \geqq 0. \end{cases}$$

*Then a necessary and sufficient condition for the single-input system* $\Sigma = (F, g, -)$ *to be positively controllable at time* $t$ *is that*

$$[(z, \exp(F(t-s)g)]_- \not\equiv 0, \quad 0 \leqq s \leqq t,$$

*for all vectors* $z$, $\|z\| = 1$.

COROLLARY *A single-input system* $\Sigma = (F, g, -)$ *is positively controllable at time* $t$ *if and only if*

*a) The matrix* $\mathscr{C} = [g|Fg|\cdots|F^{n+1}g]$ *is nonsingular.*

*b) F has no real characteristic roots.*

*c) The time* $t$ *is sufficiently large.*

The above result shows the very interesting property that no single-input system of odd order can ever be positively controllable. Also, we see that a single-input system cannot, in general, be positively controllable in the strict sense that any initial state may be transferred to the origin in arbitrarily short time by inputs from $\Omega^+$.

The multiple-input case is somewhat more delicate, requiring a surprising amount of analysis to settle with any degree of finality. A reasonable general version of the principal result is

THEOREM 7[18] *Let the input space* $\Omega$ *satisfy the conditions.*

i) $\Omega$ *contains a vector in the kernel of* $G$,

ii) *the convex hull of* $\Omega$ *has nonempty interior in* $R^m$.

*Then the following conditions are necessary and sufficient for the null-controllability of* $\Sigma$ *using inputs from* $\Omega$:

1) *rank* $\mathscr{C} = n$,

2) *there exists no characteristic vector* $v$ *of* $F'$ *satisfying* $(v, Gu) < 0$ *for all* $u \in \Omega$.

*Remark* It is important to note that the above result does not imply that the system may be driven to the origin in arbitrarily short time, only in *some* finite time. Arbitrary interval null-controllability requires substantially more restrictive assumptions of $\Omega$ than those given above. Details may be found in Ref. 28.

Now we move on to the problem of constructibility and its counterpart, observability. Roughly speaking, the problems of observability/constructibility are concerned with state determination given information about future or past outputs. We shall distinguish two kinds of state determination problems:

i) the *observation* problem, where the current state $x(\tau)$ is to be determined from knowledge of future outputs $\{y(s), s \geqq \tau\}$;

ii) the *reconstruction* problem, where the current state $x(\tau)$ is to be determined from knowledge of past outputs $\{y(s), s \leqq \tau\}$.

In the first case, we observe future effects of the present state and try to determine the cause. In the second, we attempt to reconstruct the present state without complete knowledge of the state transitions.

The basic mathematical definitions for observability/constructibility have been given above. Hence, we may pass immediately to the results. As for reachability, we begin with consideration of a constant, linear system $\Sigma = (F, -, H)$

$$\dot{x} = Fx,$$

$$y = Hx.$$

Here, of course, we set $G = 0$ since inputs play no essential role in observability questions. The main theorem is

THEOREM 8[19] *If* $\Sigma = (F, -, H)$ *is a constant linear system, then* $\Sigma$ *is completely observable/constructible if and only if the matrix*

$$\theta = [H' \,|\, F'H' \,|\cdots|\, (F')^{n-1}H']$$

*has rank* $n$.

*Remark* The observable/constructible states do not form a subspace. Just the opposite is the case: the *un*observable/*un*constructible states form a subspace of $R^n$. Thus, if rank $\theta < n$, it is not true that the linearly independent columns of $\theta$ generate the observable/constructible region.

The case of time-varying $\Sigma$ bears strong analogy to the earlier results on controllability/reachability. Specifically, we have

THEOREM 9[2] *Let* $\Sigma=(F(\cdot),-,H(\cdot))$ *be a real, continuous time, linear dynamical system. Then an event* $(\tau,x)$ *is*

a) *unobservable if and only if* $x\in\ker\hat{M}(\tau,t)$ *for all* $t>\tau$, *where*

$$\hat{M}(\tau,t)=\int_\tau^t\Phi_F'(s,\tau)H'(s)H(s)\Phi_F(s,\tau)ds;$$

b) *unconstructible if and only if* $x\in\ker M(s,\tau)$ *for all* $s<\tau$, *where*

$$M(s,\tau)=\int_s^\tau\Phi_F'(\sigma,\tau)H'(\sigma)H(\sigma)\Phi_F(\sigma,\tau)d\sigma$$

Here, again, $\Phi_F(\cdot,\cdot)$ denotes the transition matrix of $F(\cdot)$.

## B. *Linearized Systems*

Armed with the above results concerning the reachability/constructibility of linear dynamical systems, it is possible to begin to tackle various nonlinear problems. The most direct approach is to linearize the nonlinear system about a nominal control-state pair and to then apply the above linear theory for a *local* analysis.

DEFINITION 6 Consider the process

$$\dot{x}=f(x,u,t)=Fx+Gu+\cdots$$
$$y=h(x,t)=Hx+\cdots$$

near $x^*=0$, $u^*=0$ (here we use the hypotheses $f(0,0,t)=h(0,t)=0$, $(f,h$ analytic in all arguments).

The process is *locally controllable* if for each $\bar{x}$ in some neighborhood of the origin, there exists a piecewise continuous control $u(t)$, $0\leqq t\leqq\tau$, such that the system may be transferred to the state $\bar{x}$ from the origin in time $\tau$, $\tau$ sufficiently small.

The process is *locally observable* if for each sufficiently small piecewise-continuous control $u(t)$ on $0\leqq t\leqq\tau$, the equality

$$h(x_1(t),(t)=h(x_2(t),t)$$

implies

$$x_1(t)=x_2(t),\ 0\leqq t\leqq\tau,\qquad \tau \text{ sufficiently small.}$$

The basic result on local controllability and observability is that the *global* linear results are *sufficient* for the local nonlinear results, i.e.,

THEOREM 10[29] *The process* (3) *is locally controllable if*

$$\operatorname{rank}[G\,|\,FG\,|\cdots|\,F^{n-1}G]=n;$$

*it is locally observable if*

$$\operatorname{rank}[H'\,|\,F'H'\,|\cdots|\,(F')^{n-1}H']=n$$

An extensive discussion of linearized systems from the view taken here is the fundamental paper 29, which also treats the problem of when local results extend to global controllability.

## C. *Nonlinear Controllability*

As noted in the introduction, the results outlined above for finite-dimensional linear systems have now reached a rather definitive level of development and there are unlikely to be any significant changes in the basic theory in future years. When we turn our attention to nonlinear problems the situation takes on an entirely different hue. While some significant results have been obtained for certain classes of problems, there is nothing available which even hints at a comprehensive theory. Furthermore, the existing results require a much higher level of mathematical sophistication to both understand and effectively use than do those for the linear theory. For this reason, we shall not present the most general possible results for the nonlinear case in this paper, but shall be content with treating those classes of problems for which the results are particularly easy to understand and/or for which effective computational approaches seem feasible.

Extensive examination of the literature on nonlinear controllability/observability quickly suggests that a combination of algebraic and geometric techniques seems to offer the best prospects for formulating a reasonably complete theory of the structure of nonlinear problems. Thus, in the following sections we shall outline the main results after a short introductory section to set the mathematical stage for what follows. The basic concepts we shall introduce, while familiar to

workers in differential geometry, may be new to system analysts.

For sake of exposition, we first consider nonlinear systems of the form

$$\frac{dx}{dt}=\sum_{i=1}^{m} u_i(t)f_i(x), \quad x(0)=x^0, \tag{4}$$

where, as before, it is assumed that $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are *analytic* functions of their arguments. We shall consider a more general case later. The main result on the controllability of (4) is a consequence of an old theorem of Chow, for which we need the following definitions.

DEFINITION 7 Let $\{f_i\}$ be a collection of analytic vector fields on a manifold $M$. Then we say that $\{f_i\}$ is a *Lie algebra of vector fields* if

i) $\{f_i\}$ is a real vector space with respect to ordinary vector addition and scalar multiplication;

ii) if $f_j$ and $f_k$ are in $\{f_i\}$, then the Lie bracket $[f_j, f_k]$ also belongs to $\{f_i\}$. (Recall: the Lie bracket of two vector fields $f_1(x)$ and $f_2(x)$ is defined as

$$[f_1, f_2]=\left(\frac{\partial f_2}{\partial x}f_1-\frac{\partial f_1}{\partial x}f_2\right).$$

We denote the smallest Lie algebra of vector fields containing $\{f_i\}$ by $\{f_i\}_{LA}$.

DEFINITION 8 Let diff($M$) denote the group of diffeomorphorisms of $M$. Then the smallest subgroup of diff($M$) which contains $\exp tf$ for all $f$ in $\{f_i\}$ is denoted by $\{\exp\{f_i\}\}_G$. (*Note:* given $f$, for each $t$, $(\exp tf)M$ defines the map of $M\to M$ produced by the flow on $M$ given by the integral curves of the differential equation $\dot{x}=f(x)$. The field $f$ is *complete* if $(\exp tf)x$ is defined for all $|t|<\infty$).

The importance of the "exponentiating" of a vector field $f$ is that any point $x\in M$ of the form $x=\{\exp\{f_i\}\}_G x^0$ can be reached from $x^0$ because

$$x=(\exp t_1 f_{\alpha_1})(\exp t_2 f_{\alpha_2})\cdots(\exp t_m f_{\alpha_m})x^0$$

and we can use piecewise constant controls to effect the transfer.

The main question answered by Chow's theorem is: what is the difference between $\{\exp\{f_i\}\}_G x^0$ and $\{\exp\{f_i\}_{LA}\}_G x^0$? The first is clearly reachable from $x^0$, while the second seems larger. Under weak conditions, Chow's result asserts that the two are really the same.

THEOREM 11 (*Chow*) *Let the collection* $\{f_i\}_{LA}$ *be complete. Then, given any point* $x^0\in M$, *there exists a maximal submanifold* $N\subset M$ *containing* $x^0$ *such that*

$$\{\exp\{f_i\}\}_G x^0=\{\exp\{f_i\}_{LA}\}_G x^0=N.$$

This theorem gives a rather complete account of the reachability problem for systems of the form (4). The reachable set from $x^0$ is $N$ and it is constructed by exponentiating the collection of vector fields $\{f_i\}$. Furthermore, as noted above, we may reach any point $x\in N$ by application of piecewise constant controls.

The main problems with Chow's theorem are that it fails to distinguish between positive and negative time (i.e. there may be points in $N$ which can only be reached by passing from $x_0$ in the direction of negative time) and it includes no provision for a "drift" term, i.e. for systems of the form

$$\frac{dx}{dt}=g(x)+\sum_{i=1}^{m} u_i(t)f_i(x), \quad x(0)=x_0.$$

There are two approaches one can take to attack the above difficulties:

i) impose additional conditions on the vector field $\{g,f_i\}$ in order to either "factor out" or neglect the drift term $g$. Examples of both approaches are given in Ref. 30.

ii) weaken the notion of controllability. Since the second course leads to somewhat more promising general results, we sketch the main results from Ref. 31. We begin with a notion of *local* accessibility.

DEFINITION 9 Given a subset $U\subseteq M$, we say that $x^1$ is *U-accessible* from $x^0$ (denoted $x^1 A_u x^0$) if there exists a bounded, measurable control $u(t)$, $t_0\leqq t\leqq t_1$, such that the solution of the differential equation

$$\dot{x}=f(x,u)$$

satisfies $x(t_0)=x^0$, $x(t_1)=x^1$ and $x(t)\in U$ for all $t_0 \leqq t \leqq t_1$.

The idea of U-accessibility allows us to weaken the previous definition of controllability in the following manner.

DEFINITION 10 Given an open set $U\subseteq M$, there is a smallest equivalence relation $WA_U$ on $U$ which contains all $U$-accessible pairs. If $U=M$, then we write the relation as $WA$ and say that the system is *weakly controllable* at $x^0$ if $WA(x^0)=M$.

The advantage of weak controllability is that it admits of a simple algebraic test. We adopt the following notation to describe the result.

Consider the differential equation

$$\dot{x}=f(x,u)$$

and let $u^i$ be a *constant* control law. Such a law generates the vector field $f^i \doteq f(x,u^i)$. As before we let $\mathscr{F}=\{f_i\}_{LA}$ denote the Lie algebra generated by the vector field, $\{f_i\}$. Finally, we have

DEFINITION 11 The system is said to satisfy the *controllability rank conditions* at $x^0$ if $\mathscr{F}(x^0)$ has dimension $n$.

The relevance of the preceding definition for controllability is shown in the next result.

THEOREM 12[31] *If the system $\Sigma$ is analytic, then it is locally controllable if and only if the controllability rank condition is satisfied for all $x\in M$.*

*Example* We apply the foregoing result to study controllability of the single-input bilinear system

$$\dot{x}=Ax+u(t)Bx, \quad x(0)=x^0.$$

Here, the space $M=R^n$.

It is easy to verify that the Lie bracket of the linear vector fields $Ax$, $Bx$ is

$$[Ax,Bx]=[A,B]x$$
$$=(BA-AB)x$$

Thus, $\mathscr{F}$ is isomorphic to a subgroup of $gl(n\,R)$, the set of $n\times n$ real matrices. Let

$$F=\{\exp X : X\in\mathscr{F}\}$$

i.e. $F$ is obtained by exponentiating $\mathscr{F}$, as discussed above. We can conclude by Theorem 12 that the bilinear system above is weakly controllable if and only if $\mathscr{F}(x^0)$ has dimension $n$, i.e., if the subgroup of $gl(n\,R)$ formed by $A$, $B$ and all Lie brackets of $A$ and $B$ has dimension $n$. In geometric terms, this is the algebraic equivalent of saying that $F$ acts transitively on $M=R^n$.

*Example* A somewhat less trivial example is provided by the following simple water pollution problem. We have a lake in which three component items are of interest: nutrient, phytoplankton and zooplankton. Denoting these as $z_1$, $z_2$, $z_3$, respectively, the postulated dynamics are:[32]

$$\dot{z}=Az+b(z)+Gu(t)+f(t)$$

where

$$A=\mathrm{diag}(-d,-d,-d), \quad G=\begin{pmatrix}1\\0\\0\end{pmatrix},$$

$$b(z)=\begin{bmatrix}\left(\frac{-\mu_1}{m_1}\right) & \frac{z_1z_2}{k_1+z_1} & \\ \left(\frac{-\mu_2}{m_2}\right) & \frac{z_2z_3}{k_2+z_2}+\frac{\mu_1z_1z_2}{k_1+z_2} & \\ & \frac{\mu_2z_2z_3}{k_2+z_3} & \end{bmatrix},$$

$$f(t)=\begin{bmatrix}ds_0\\0\\0\end{bmatrix},$$

where $\mu_1$, $\mu_2$, $k_1$, $k_2$, $s_0$ and $d$ are positive constants representing reproduction rates, Michaelis constants, flow velocity and initial nutrient level, respectively. The dynamics are such that it is possible to directly influence only the nutrient level by controlling action.

Since we are concerned only with physically meaningful population levels of the three components of $z$, the relevant space $M$ is

$$M=\{z\in R^n : z_i\geqq 0, \quad i=1,2,3\}$$

i.e. $M$ is the positive orthant in $R^3$. The system is clearly analytic in this region due to the positivity assumptions on $m_1,m_2,k_1,k_2$. In addition, to study controllability we can assume without

loss of generality that $f(t) \equiv 0$, by the usual translation of coordinates argument.

To apply the foregoing theorem to this problem, we must compute the vector fields $\{Az + b(z), G\}$ and their various Lie brackets. We have

$$[G, G] = 0,$$

$$[G, Az + b(z)] = \begin{bmatrix} -d - \dfrac{\mu_1 k_1 z_2}{m_1 (k_1 + z_1)^2} \\ \dfrac{\mu_1 z_2}{k_1 + z_2} \\ 0 \end{bmatrix}$$

$$[G, [G, Az + b(z)]] = \begin{bmatrix} \dfrac{2\mu_1 k_1 z_2}{m_1 (k_1 + z_1)^3} \\ 0 \\ 0 \end{bmatrix}$$

$$[Az + b(z), [G, Az + b(z)]] = \begin{bmatrix} x \\ x \\ \dfrac{-\mu_1 \mu_2 z_3 z_2}{(k_2 + z_3)(k_1 + z_2)} \end{bmatrix},$$

where "$x$" denotes an unwieldly non-zero algebraic quantity.

The above results show that if $\mu_1 \mu_2 \neq 0$, the Lie algebra $\{Az + b(z), G\}_{LA}$ has dimension $3 = \dim R^3$. Hence, the controllability rank condition is satisfied and the system is weakly controllable from any initial state $z^0$ such that $z_2^0 \neq 0$, $z_3^0 \neq 0$.

On the basis of the foregoing, we conclude that as long as the reproduction rates of the phyto- and zooplankton remain positive, it is possible to drive the system to the origin by controlling the nutrient level in the lake. While this result is not too hard to derive directly from the dynamics, the relative ease with which it is obtained from Theorem 12 suggests use of the Lie-algebraic techniques for more complicated water resource systems.

### D. *Nonlinear Observability/Constructibility*

The observability/constructibility problem for nonlinear systems involves, in general, an even greater level of mathematical complication than the controllability results just discussed. As wit controllability, the essence of the observabilit question is geometric and a reasonably complet treatment of the general case involved "dualizing the Lie-algebraic controllability concepts throug the algebra of differential forms. Since the techn cal preliminaries needed to present such resul would take us too far beyond the bounds of th report, we suggest Ref. 31 for complete details. I this section, we restrict our results to speci cases which can be easily treated computationall and which present features of practic importance.

The first problem of interest is bilinear dy namics with *linear* observations. The system described by the equations

$$\frac{dx}{dt} = Fx + Gu + \sum_{i=1}^{m} u_i N_i x, \quad y(t) = Hx(t).$$

The result governing observability of such process is

THEOREM 13[20] *Define the matrix sequence* $\{Q$ *by the recurrence formulas*

$$Q_1 = H, \quad Q_i = \begin{bmatrix} Q_{i \ \ 1} F \\ \hline Q_{i \ \ 1} N_1 \\ \hline \vdots \\ \hline Q_{i-1} N_m \end{bmatrix}, \quad i = 1, 2, \ldots, n.$$

*Then the subset of unobservable states of th system is the null space of the matrix*

$$\begin{bmatrix} Q_1 \\ \hline Q_2 \\ \hline \cdots \\ \hline Q_n \end{bmatrix}.$$

Thus, we see that purely linear algebraic com putations suffice to characterize the *un*observabl subspace of $R^n$ for a bilinear system possessin linear observations.

Now we consider the more general case whe the output, as well as the dynamics may b nonlinear, i.e. the system is described by th equations

$$\frac{dx}{dt} = f(x, u), \quad x(0) = x^0, \quad y(t) = h(x, t).$$

DEFINITION 12 We say the above system is *completely observable on* $[0, T]$ if there exists a 1–1 correspondence between $X_0$, the set of initial states, and the set of observed output functions $y(t)$, $0 \leqq t \leqq T$.

It is convenient to define the auxiliary variables

$$z(0) = \begin{bmatrix} y(0) \\ \dot{y}(0) \\ \vdots \\ y(k-1)(0) \end{bmatrix}, \quad H(x^0) = \begin{bmatrix} h_0(x^0, 0) \\ h_1(x^0, 0) \\ \vdots \\ h_{n-1}(x^0, 0) \end{bmatrix},$$

where

$$h_{k-1}(x^0, 0) = \frac{\partial h_{k-2}(x^0, 0)}{\partial t} + \frac{\partial h_{k-2}}{\partial x}(x^0, 0) f(x^0, 0), \quad k = 1, 2, \ldots, n.$$

We note that the injectivity of the map $H: X_0 \to H(X^0)$ provides a sufficient condition for *global* observability of the system. In general, though, it is not necessary since $z(0)$ doesn't represent the entire output $y(t)$, $0 \leqq t \leqq T$.

The main result enabling us to decide whether or not the map $H$ is 1–1 is

THEOREM 14[25] *Let $J(x)$ be the Jacobian map of $H$ at $x$. If there exists an $\varepsilon > 0$ such that the absolute values of the leading minors of $J$ satisfy*

$$|\Delta_1| \geqq \varepsilon, \quad \left|\frac{\Delta_2}{\Delta_1}\right| \geqq \varepsilon, \ldots, \left|\frac{\Delta_n}{\Delta_{n-1}}\right| \geqq \varepsilon,$$

*for all $x \in R^n$, then $H$ is 1–1 on $X_0$.*

*Example* Consider the system

$$\dot{x}_1 = \tfrac{1}{2}x_1^2 + e^{x_2} + x_2,$$
$$\dot{x}_2 = x_1^2$$
$$y(t) = x_1(t).$$

We have

$$H(x) = \begin{pmatrix} x_1 \\ \tfrac{1}{2}x_1^2 + e^{x_2} + 1 \end{pmatrix},$$
$$J(x) = \begin{bmatrix} 1 & 0 \\ x_1 & e^{x_2} + 1 \end{bmatrix}.$$

Thus, the conditions of the theorem are satisfied with $\varepsilon = 1$ and the system is globally observable.

### E. *Minimality, Canonical Structures and Realizations*

While it is not really within the purview of this paper to enter into a thorough discussion of realization theory for nonlinear processes, reachability and observability play such a central role in determining the existence and properties of such realizations that we would be remiss if the basic results were not at least mentioned.

It is well known that a linear system has a state-space realization of minimal dimension if and only if the system is both completely reachable and completely observable. Furthermore, any two systems initialized at 0 which minimally realize the same input/output map differ only by a linear coordinate transformation of the state space. The question naturally arises as to how (if at all) these results extend to the nonlinear case. As noted, the general case is much beyond the scope of this review and the reader is referred to [33] for a thorough treatment. However, for the case considered above of *analytic* systems, we have the following version of a result due to Sussman.

THEOREM 15[33] *Let $\Sigma, \Sigma^1$ be two analytic systems which are observable, weakly controllable and which realize the same input/output map from their respective initial states. Then $\Sigma$ and $\Sigma^1$ differ only by a diffeomorphism of their state spaces.*

## III. APPLICATION AREAS

This section examines several problems that have been described in earlier publications from the viewpoint of the methodology discussed above. Many of the problems that follow were originally presented within the context of an optimization process. In accordance with the general principle that system structure is more or less independent of externally imposed criteria, we abstract here only those features which do not relate to the particular objective function initially chosen.

A. *Renewable Resources Management*

A major ecological problem is to develop strategies for the development of resources. The real issue here, as pointed out in Refs. 8, 21, is to devise sensible (and implementable) harvesting policies in the face of uncertainties regarding the dynamics of the process and within the context of an everchanging political and economic environment. This question properly belongs within the domain of adaptive control theory; however, various versions of it may be considered as reachability/constructibility questions for which the above methodology may yield some insight.

Suppose we have a resource system whose state at time $t$ may be described by a population function $x(t)$. A simplified version of the dynamics of this system as adapted from Ref. 21 is

$$\dot{x}(t) = F(x) - aE(t)x(t),$$

Here $F(x)$ is a function representing the natural growth rate of the population, $a$ is a "catchability" coefficient, and $E(t)$ is a function representing the harvesting effort (control) or some other measure of exploitation intensity.

The function $F(x)$ may take many forms depending upon the particular type of ecosystem under study. Probably the most prevalent is the logistics growth form

$$F(x) = rx(1 - x/K),$$

where

$r$ = the net proportional growth rate of the population

$K$ = the environmental carrying capacity or saturation level.

Choosing $F(x)$ of the logistics form, the process is quadratically nonlinear in the state, with control entering bilinearly; however, if the saturation level is very high as compared with $x$, i.e. $K \gg x$, then we may well approximate the process as a bilinear system, homogeneous in the state.

Other forms of $F(x)$ that have been proposed include

$$F(x) = \begin{cases} rx, & x < K \\ -\infty, & x > K, \end{cases}$$

and

$$F(x) = mx + b, \quad m, b \text{ constants.}$$

Clearly, both of these forms would yield a ecosystem well within the bounds of the theor presented earlier.

Of course, the elementary scalar proble above may be dealt with by any number techniques. It is only when we pass to th multispecies version, as discussed in Ref. 21 tha the mathematical apparatus of this paper wi truly show its merit.

Problems of observability/constructibility als enter into the above framework. In fact, accord ing to Ref. 8: "in most cases $x(t)$ is not directl observable... usually it is only possible to mea sure the total catch $aE(t)x(t)$ and the effort $E(t)$. Thus, we see that the observability technique may provide essential information to the syste manager in enabling him to accurately measur the population level of the system before decidin upon his action $E(t)$. Alternatively, system theoretic techniques may show the manager tha there are inherent structural features of the sy tem that will forever prevent him from obtainir complete information about the process, even i the *absence* of stochastic effects. Such knowled may suggest alternate models and/or a restructu ing of the basic measurement process.

In many instances, some or all of the com ponents of the system parameters $a, r$, and $K$ ma be subject to stochastic perturbations havin known or unknown distribution functions. In th first case, the preceding methodology may b used in an expected value sense; in the latte case, one is faced with an adaptive process whos methodological treatment transcends the lim tations and modest aims of this report. The ke point, however, is that the introduction of ran domness into the process may complicate th computational aspects of the situation but presents no new conceptual hurdles. Thus, thorough understanding of the deterministi methodology will, with sufficient computing pow er, enable one to deal also with the stochasti case.

A somewhat more complicated version of th above problem is given in Ref. 9 for the de termination of harvesting strategies for salmo

n this problem, we have the dynamics (after ome algebraic re-arrangement)

$$S_{t+1} = \exp(-\alpha(1-S_t))S_t z_t$$

vhere $S_t$ is the salmon stock level at time $t$, $\alpha$ is a parameter reflecting the net stock productivity, nd $z_t = 1/(1-u_t)$, with $u_t$ representing the net xploitation rate.

By expanding the exponential term as

$$\exp(-\alpha(1-S_t)) \cong 1 - \alpha(1-S_t) + \frac{\alpha^2}{2}(1-S_t)^2 + \ldots,$$

nd truncating at some appropriate point, the almon model becomes a polynomial system with ontrol entering linearly. The reachability struc- ure of such systems may be studied by several neans: linearization, conversion to a multilinear roblem by introduction of additional state vari- bles, or directly by Lie-algebraic methods.

### 3. *Water Resource Problems*

A problem that arises in most water basin net- vorks throughout the world is that of regulating he flow through various dams in the network in rder that the entire system behave in some rescribed fashion. Generally, this problem is omplicated by the presence of stochastic inflows o the network due to rainfall and underground vater run-off. In addition, the conflicting objec- ives of the various water users must be taken nto account. Here we shall consider only the eachability/constructibility questions involving he physical water basin network itself.

A simple example of a problem of this genre aken from Ref. 10 is depicted in Figure 2. In igure 2, $r_1(t)$ and $r_2(t)$ are the rainfall inputs, ne states of surface storage at locations 1–3 are $_1(t)$, $x_2(t)$, $x_3(t)$, respectively, while the state of roundwater storage (including infiltration) is $_4(t)$. The constants $l_1$ and $l_2$ are for infiltration. he expression $l_3(x_4 - x_3)$ signifies the exchange etween the river and groundwater. The system utputs $y_1$, $y_2$ are the streamflow output and the ontribution of groundwater, respectively.

The continuity equations for this problem are

$$\dot{x}_1 = -l_1 x_1 + r_1 - g_{11}u_1,$$
$$\dot{x}_2 = -l_2 x_2 + r_2 - g_{22}u_2,$$
$$\dot{x}_3 = l_3(x_4 - x_3) - kx_3 + g_{11}u_1 + g_{22}u_2,$$
$$\dot{x}_4 = l_1 x_1 + l_2 x_2 - l_3(x_4 - x_3).$$

The outputs are

$$y_1 = kx_3,$$
$$y_2 = l_3(x_4 - x_3).$$

In vector-matrix form, we have

$$\dot{x}(t) = Fx + Gu + r(t), \quad x(0) = c,$$
$$y(t) = Hx,$$

where

$$F = \begin{bmatrix} -l_1 & 0 & 0 & 0 \\ 0 & -l_2 & 0 & 0 \\ 0 & 0 & -(k+l_3) & l_3 \\ l_1 & l_2 & l_3 & -l_3 \end{bmatrix}$$

$$G = \begin{bmatrix} -g_{11} & 0 \\ 0 & -g_{22} \\ g_{11} & g_{22} \\ 0 & 0 \end{bmatrix}, \quad r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ 0 \\ 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 & k & 0 \\ 0 & 0 & -l_3 & l_3 \end{bmatrix}.$$

By virtue of the closed form expression

$$x(t) = \exp(Ft)c + \int_0^t \exp(F(t-s))[Gu(s) + r(s)]ds,$$

we see that the reachability/constructibility features of the above process are independent† of the rainfall input $r(t)$. Thus, for purposes of analysis, there is no loss of generality in assuming $r(t) \equiv 0$. (Intuitively, this is due to the tacit assumption that $u(t)$ can be made arbitrarily large. In the more realistic case when $0 \leqq u_i(t) \leqq U$, a more refined analysis is required.)

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†Independent in the sense that results for the case $r(t) \not\equiv 0$ may be obtained from the $r = 0$ case simply by adding the known vector function $-\int_0^t \exp(F(t-s))r(s)ds$ to $x(t)$.

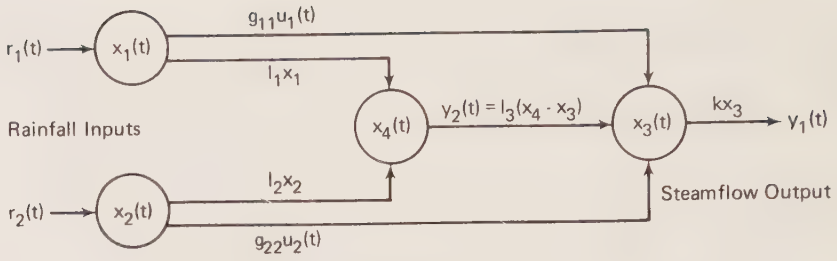


FIGURE 2

It is a simple exercise to apply the techniques of Part II to the above system to discover what is already evident from Figure 2; namely, that the system is completely reachable as long as $g_{11} \neq 0$, $g_{22} \neq 0$, and is also completely constructible. However, note the central role played by groundwater interchange with surface storage in the determination of constructibility, i.e., if $l_3 = 0$ (no interchange) then no amount of streamflow observation gives any information about groundwater storage.

As another example from the water resource area where system structure plays an important role, we consider the problem of parameter determination in a rainfall-runoff process. Here we shall be able to utilize results on nonlinear observability to answer some important practical problems concerning system structure.

Direct runoff may be considered as the result of rainfall excess being transformed by a basin system. One model proposed in Ref. 26 for the storage equation is

$$S(t) = K_1 Q^N(t) + K_2 \frac{dQ}{dt},$$

where

$S(t)$ = water storage at time $t$ in the basin,

$Q(t)$ = direct water runoff at time $t$,

$K_1, K_2, N$ = parameters characterizing the particular basin.

Using the continuity equation, the following differential equation for the rainfall runoff is obtained

$$K_2 \frac{d^2Q}{dt^2} + K_1 N Q^{N-1} \frac{dQ}{dt} + Q = U,$$

Where $U(t)$ is the excess rainfall at time $t$. T basic problem is to determine the paramet $K_1, K_2$, and $N$, given observations of the dir runoff.

Mathematically, we may model the above p cess by the nonlinear system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -x_3 x_4 x_5 x_1^{x_5 - 1} x_2 + x_4(U - x_1),$$

$$\dot{x}_3 = 0,$$

$$\dot{x}_4 = 0,$$

$$\dot{x}_5 = 0,$$

where

$$x_1 = Q,\ x_2 = \dot{Q},\ x_3 = K_1,\ x_4 = 1/K_2,\ x_5 = N.$$

The observation equation is

$$y(t) = x_1(t).$$

Results on observability of nonlinear systems now enable us to determine whether or observations of water runoff alone are suffici to determine the parameters $K_1$, $K_2$, and $N$.

Other water resource problem of a sim nature may be found in the Refs 11 and 12.

### C. *National Settlement Planning*

A number of urbanologists have been concern with the question of developing national set ment strategies subject to constraints on sources, immigration quotas, and the like. Seve different approaches have been proposed for de ing with this sort of problem, some of th falling into the basic framework considered

is paper. We describe one of these "system-eoretic" approaches, first presented in Ref. 13. he essential aspect of this model is to promote a esired migratory process by differential stimu-tion of the employment market on the part of ie government.

The state equations for the model are

$$x(t+1)=Kx(t)+(I-M)v(t),$$

$$v(t+1)=Mv(t)+u(t)+z(t),$$

here the vector $x(t)\in R^n$ represents the popu-tion distribution at time $t$, $v(t)\in R^n$ is the distri-ution of job vacancies at time $t$, $u(t)\in R^n$ is the istribution of government stimulated job vacan-es and $z(t)\in R^n$ is the distribution of spon-neously occurring vacancies. The matrix $K$ is a iagonal matrix whose elements reflect the na-ural population growth rates within a region, hile $M$ is a migration matrix with elements $m_{ij}$ eing the probability that a job vacancy in region will be filled by someone living in region $i$, $i,j = 1,\ldots,n$. The problem, of course, is to choose $t)$ so that $x(t)$ (and possibly $v(t)$) follows some esired course.

The budgetary and fixed immigration con-raints on the choice of $u(t)$ are given by

i) $u(t)\geqq 0$,

ii) $(u(t), r(t))\leqq b$,

iii) $\|u(t)\|\leqq \hat{u}, \quad t=1,2,\ldots,T.$

ere $(,)$ denotes the vector inner product, while $\cdot\|$ is some appropriate norm (e.g., $l_1$), with $r(t)$ eing a function giving the total financial re-ource available to be offered regionally by the overnment at period $t$, $b$ being the total budget vailable.

By introducing the new vectors

$$s(t)=\begin{bmatrix}0\\ \hline u(t)\end{bmatrix}, \quad w(t)=\begin{bmatrix}x(t)\\ \hline v(t)\end{bmatrix},$$

$$y(t)=\begin{bmatrix}0\\ \hline z(t)\end{bmatrix}, \quad s,w,y\in R^{2n},$$

is possible to rewrite the above model in the rm

$$w(t+1)=Fw(t)+Gs(t)+y(t),$$

where

$$F=\left[\begin{array}{c|c} K & I-M \\ \hline 0 & M \end{array}\right], \quad G=\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & I \end{array}\right].$$

The above constraints restrict the region of admissible inputs $s(t)$. Actually, on the basis of more detailed analysis, for purposes of determining reachable sets it suffices to replace inequalities (ii) and (iii) by the corresponding equality. (Physically, this fact is fairly obvious but requires a surprising amount of analysis to prove.)

The usual questions surrounding reachability may now be studied with the above "standard" model. As with the previous water example, the forcing term $y(t)$, corresponding to the spontaneously arising jobs, plays no structural role in the reachability analysis.

The model given in Ref. 13 included no inherent notion of observability. However, it is reasonable to suppose that in many cases the vectors $x(t)$, $v(t)$ will not be available for direct measurement and, if the migration strategy is to be based upon the current state of the system rather than upon an "ostrich-like" open-loop policy, then observability/constructibility questions will arise. A more detailed pursuit of this point will likely prove necessary before a model of the above type could be used in practice with any degree of confidence.

### D *Transportation Systems*

Problems involving the regulation of automotive traffic flow in urban areas seem tailormade for the type of methodology we have been discussing. Many, if not most, of the mathematical models surveyed in Ref. 14 involve systems in which the dynamics of the traffic flow are linear, with the primary question being whether or not it is possible to reach a state of "undersaturation" (normal flow), from a state of "oversaturation" (rush-hour congestion) within a specified time period utilizing various regulation policies (usually control of freeway on-off ramps by traffic lights). Obviously, this is a reachability problem. If the control policy is to be generated by

feedback, then observability/constructibility considerations enter when one analyzes the possibility of determining the current state of traffic based upon measurements being received from various sensing devices. Here we present one of the examples from Ref. 14 to illustrate the main ideas.

Consider the rectangular traffic network depicted in Figure 3. We assume that the network is oversaturated; i.e., at one or more intersections traffic demand exceeds capacity. Let $x_i(t)$ be the number of cars waiting at intersection $i$, and let $u_i(t)$ denote the number of cars leaving intersection $i$ during the green light. If we assume th the travel time between two intersections is sm compared to the waiting time, then the dynam of the process are reasonably well described the equations

$$x(t+1)=x(t)+Gu(t)+q(t),$$

where the vector $q(t)$ has components $q_i(t)$ repr senting the external traffic arriving at intersecti $i$ during period $t$. It is clear from Figure 3 th the flows $u_3$, $u_6$, $u_9$, and $u_{10}$ are flows out of t network.

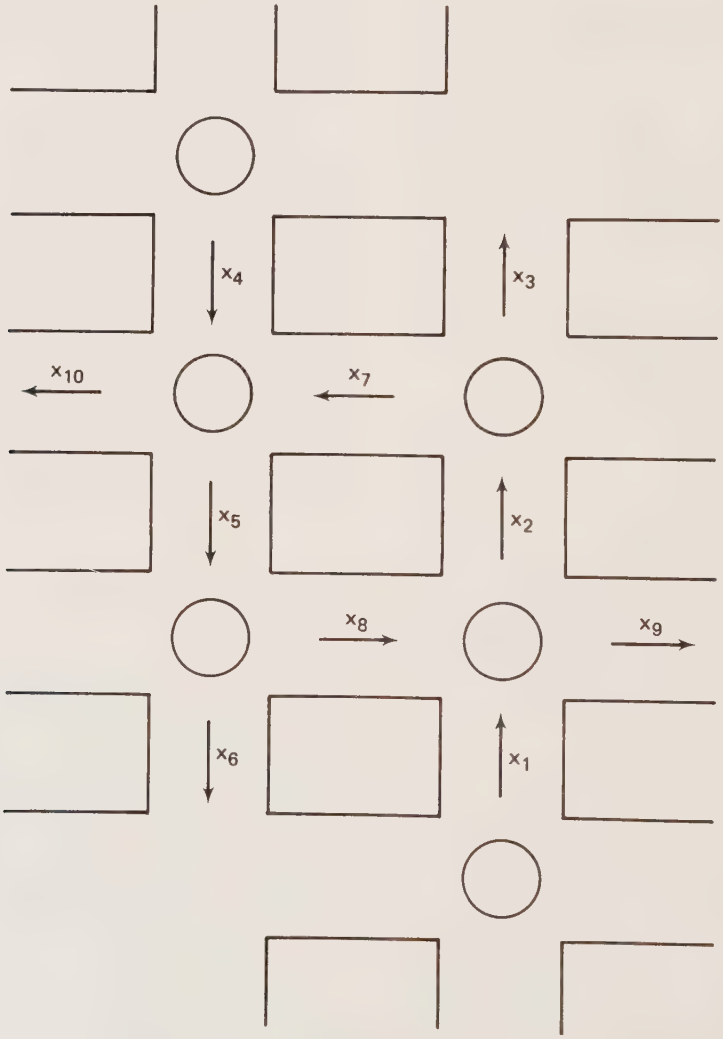


FIGURE 3

The control matrix $G$ takes the form

$$G = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ s_1 & -1 & 0 & 0 & 0 \\ 0 & s_2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & s_4 & -1 \\ 0 & 0 & 0 & 0 & s_5 \\ 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_5 \\ r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & r_7 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & s_8 & -1 & 0 \\ 0 & s_7 & 0 & 0 & -1 \end{bmatrix}$$

The elements $r_i$ and $s_i$ denote the percentage of cars turning right or left ($r_i$) and going straight ahead ($s_i$).

On psychological grounds, it is reasonable to impose the control constraints

$$M_i \leqq u_i(t) \leqq U_i \quad i = 1, \ldots, 10.$$

where $M_i$ and $U_i$ represent the minimal and maximal number of cars that are acceptable during a given green time.

The basic problem is now quite simple: given an initial state $x(0)$, assumed to be an oversaturated condition, is there a control policy $u(t)$ which transfers $x(t)$ to an undersaturated region within a prescribed time $T$? This is clearly a problem of conditional controllability as discussed in section II. However, the constraints imposed on the admissible inputs mean that Theorem 1″ cannot be applied in a routine fashion. Additional work is required to suitably modify it along the lines taken to obtain the positive controllability result Theorem 6.

## REFERENCES


1. R. Kalman, P. Falb, and M. Arbib, *Topics in Mathematical Systems Theory*. McGraw-Hill, New York, 1969.
2. R. Kalman, "Lectures on controllability and observability." *Proc. CIME Summer School*, Edizioni Cremonese, Rome, Rome, 1969.
3. R. Kalman, "On the general theory of control systems." *Proc. 1st IFAC Congress*, Moscow, 1960.
4. R. Gabasov and F. Kirillova, *The Qualitative Theory of Optimal Processes*. Nauka, Moscow, 1969 (in Russian). (Engl. tran. Marcel Dekker, New York, 1976.)
5. R. Brockett, *Finite-Dimensional Linear Systems*. John Wiley, New York, 1970.
6. R. Brockett, "Lie algebras and Lie groups in control theory." *Proc. NATO Advanced Study Institute on Geometric and Algebraic Methods for Nonlinear Systems*, London, 1973.
7. C. Lobry, "Dynamical polysystems and control theory." *Proc. NATO Advanced Study Institute on Geometric and Algebraic Methods for Nonlinear Systems*, London, 1973.
8. C. Walters, "Adaptive control problems in renewable resource development." *Internal Paper, IIASA*, 1975.†
9. C. Walters, "Optimal harvest strategies for salmon in relation to environmental variability and uncertainty about production parameters." *Internal Paper, IIASA*, 1975.
10. A. Sz. Nagy, "State-space approach to hydrology." *Symposium on Math. Modelling in Hydrology*, University College, Galway, Ireland, April, 1974.
11. J. Casti, "Algorithms for the stochastic inflow-nonlinear objective water reservoir control problem." *IIASA Workshop on Vistula and Tisza River Basins*, IIASA, February, 1975.
12. A. Sz. Nagy, "On the optimal stochastic control of water resource systems." *Internal Paper, IIASA*, 1975.
13. R. Mehra, "An optimal control approach to national settlement system planning." *IIASA RM-75-58*, IIASA, 1975.
14. H. Strobel, "Transportation, automation, and the quality of urban living." *IIASA RR-75-34*, IIASA, 1975.
15. R. Kalman, B. Ho, and K. Narendra, "Controllability of linear dynamical systems." *Contr. to Diff. Eqs.*, **1**, 1963, pp. 189–213.
16. J. Wei and E. Norman, "On the global representation of the solutions of linear differential equations as a product of exponentials." *Proc. Amer. Math. Soc.*, **15**, 1964, pp. 327–334.
17. J. P. Charpentier, "Overview on techniques and models used in the energy field." *IIASA RM-75-8*, IIASA, 1975.


†Throughout the references, the initials IIASA stand for the International Institute for Applied Systems Analysis, Laxenburg, Austria. Most of the IIASA papers cited are available upon request from IIASA.

18. R. Brammer, "Controllability in linear autonomous systems with positive controllers." *SIAM J. Control*, **10**, 1972, pp. 339–353.
19. J. Casti, *Dynamical Systems and Their Applications: Linear Theory*. Academic Press, New York, 1977.
20. P. D'Alessandro, A. Isidori, and A. Ruberti, "Realization and structure theory of bilinear dynamical systems." *SIAM J. Control*, **1**, 1974, pp. 517–535.
21. C. Clark, *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*. Interscience, New York, 1976.
22. G. Marchesini and G. Picci, "Some results on the abstract realization theory of multilinear systems." In: *Theory and Application of Variable Structure Systems*, R. Mohler and A. Ruberti, eds., Academic Press, New York, 1972.
23. G. Haynes and H. Hermes, "Nonlinear controllability via Lie theory." *SIAM J. Control*, **8**, 1970, pp. 450–460.
24. E. Sontag, *On the Internal Realization of Polynomial Response Maps*. Ph.D. Thesis, Mathematics Dept., University of Florida, Gainesville, Florida, 1976.
25. S. Kou, D. Elliott, and T. Tarn, "Observability of Nonlinear Systems." *Infor. and Control*, **22**, 1973, pp. 89–99.
26. N. Duong, C. Winn, and G. Johnson, "Modern control concepts in hydrology." *IEEE Tran. Syst. Man, and Cyber.*, **SMC-5**, 1975, pp. 46–53.
27. S. Saperstone and J. Yorke, "Controllability of linear oscillatory systems using positive controls." *SIAM J. Control*, **9**, 1971, pp. 253–262.
28. D. Jacobson, *Extensions of Linear-Quadratic Control, Optimization and Matrix Theory*. Academic Press, New York, 1975.
29. H. Hermes, "On local and global controllability." *SIAM J. Control*, **12**, 1974, pp. 252–261.
30. R. Brockett, "Nonlinear systems and differential geometry." *Proc. IEEE*, **64**, 1976, pp. 61–72.
31. R. Hermann and A. Krener, "Nonlinear controllability and observability." *IEEE Tran. Auto Cont.*, **AC-22**, 1977, pp. 728–740.
32. A. Bazykin, "Elementary model of eutrophication." In: H. Grümm, ed., *Analysis and Computation of Equilibria and Regions of Stability*, IIASA, CP-75-8.
33. H. Sussman, "Minimal realizations of nonlinear systems." In: D. Mayne and R. Brockett, eds., *Geometric Methods in Systems Theory*, D. Reidel, Dordrecht, Netherlands, 1973.

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# COMMUNICATION WITHOUT A CHANNEL

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The dominant paradigm for communication between $A$ and $B$ involves a channel, i.e., a chain of causes and effects, from $A$ to $B$ or from $B$ to $A$. This paper shows that communication in the mathematical sense of information theory does not require a direct channel from $A$ to $B$ or from $B$ to $A$ but can result from communication to $A$ and $B$ from a common source. The importance of this form of communication is discussed.

INDEX TERMS Communication channel, transmission, mutual coordination.

## 1 INTRODUCTION

When Shannon clarified and formalized the notation of communication in his classic work. *The Mathematical Theory of Communication*,[1] he used a paradigm for communication shown in Figure 1. Shannon introduced a measure of the amount of communication between the sender $A$ and the receiver $B$; it is denoted $T(A:B)$ and is called the transmission† between $A$ and $B$. In Shannon's model the channel plays a prominent role, and he shows that $T(A:B)$, put on a per-second basis, is bounded by the "capacity" of the channel, which for any physical channel is a definite finite quantity.

So dominant has this model of communication become that for most people it is difficult to imagine communication without a channel. Let the reader consider: if there is no channel, or causal connection, from $A$ to $B$, and none from $B$ to $A$, is it possible for there to be communication between $A$ and $B$? It is the claim of this paper that such communication is indeed possible, in the technical sense that $T(A:B)$ can be positive. Not only is communication without a channel possible but it is in fact commonplace, perhaps even more important in our everyday lives than communication requiring channels.

To support this claim we will first provide a technical argument and subsequently present an interpretation of the results.

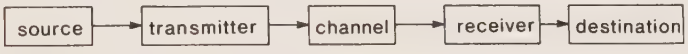


FIGURE 1

## II COMMUNICATION

The most satisfactory measure of communication between $A$ and $B$ is Shannon's measure $T(A:B)$. Its definition rests on his definition of entropy, $H$, defined thus:

$$H(X) = -\sum_i p(x_i)\log_2 p(x_i)$$

$$H(X, Y) = -\sum_i \sum_j p(x_i, y_j)\log_2 p(x_i, y_j).$$

†Also denoted $I(A;B)$ and called the transinformation or mutual information. The notation in this article is that of Ashby.[2]

The entropy of a variable $X$ is a measure of its unpredictability, and therefore of the amount of information conveyed in communicating its value. Similarly $H(X, Y)$ is a measure of the uncertainty of the joint variable $\langle X, Y \rangle$. The conditional entropy or uncertainty of $X$ when $Y$ is given or known is denoted $H_Y(X)$:

$$H_Y(X) = H(X, Y) - H(Y).$$

The transmission between $A$ and $B$ can now be defined in either of three equivalent ways, symmetrical in $A$ and $B$:

$$\begin{aligned} T(A:B) &= H(A) + H(B) - H(A, B) \\ &= H(A) - H_B(A) \\ &= H(B) - H_A(B). \end{aligned}$$

The first of these defines $T(A:B)$ as the uncertainty about the individual variables, less that about the couple. The others define $T(A:B)$ as the amount by which the uncertainty about one variable is reduced when knowledge of the other is taken into account.

It is well known that all entropies and transmissions, including conditional (subscripted) quantities, are nonnegative, and that $T(A:B)$ is zero if and only if $A$ and $B$ are probabilistically independent. $T(A:B)$ is essentially a measure of coordination of behavior of two variables, so that whenever such coordination occurs and $T(A:B)$ is positive we may say that there is communication between $A$ and $B$, since knowledge of one of the variables provides at least partial information about the other.

There may be some resistance on the part of the reader to the assertion that $T(A:B) > 0$ implies communication between $A$ and $B$, regardless of the reason for the positive transmission. Yet if $T > 0$ provides the criterion when there *is* a channel there seems no plausible reason why it should not do so when there is *not* a channel. It appears to be the only objective criterion available.

Entropies and transmissions may be put on a per-second basis by use of a limiting process. To do so here would complicate the technical argument, however, and since all identities which are true for entropies and transmissions as defined above remain true on a per-second basis,[3] there is no need for that complication. The argument holds true on either basis.

## III CHANNELS

Suppose we have a collection $\mathscr{S}$ of systems $S_i$, with some directly affecting others. We will use $C(S_i, S_j)$ as shorthand for the proposition that there is a channel from $S_i$ to $S_j$. $C(S_i, S_j)$ is true if and only if there is a continuous unidirectional chain of causality from system $S_i$ to system $S_j$, that is, either $S_i$ directly affects $S_j$ or there is an ordered set of systems $\{S_i, S_1, S_2, \ldots, S_n, S_j\}$ linking $S_i$ to $S_j$, with each member directly affecting its successor in the set. Notice that although the causal chain implies a channel, the channel may actually have a capacity of zero, for it is possible for $S_i$ to affect $S_k$, and for $S_k$ to affect $S_j$, but for $S_i$ to have no ultimate effect whatever on $S_j$. In such a case it is common to claim that there is no channel from $S_i$ to $S_j$, but it is simpler here to claim that there is a channel but with capacity zero.

By definition, $C(S_i, S_i)$ is always false.

For every system $S_j$ in $\mathscr{S}$ there is an associated input set, denoted $I(S_j)$, of systems $S_i$ in $\mathscr{S}$ such that there is a channel from $S_i$ to $S_j$:

$$I(S_j) = \{S_i : C(S_i, S_j)\}.$$

Let us consider two systems in $\mathscr{S}$ more simply labeled $A$ and $B$. We will assume that no channel exists between them:

$$A \notin I(B)$$

$$B \notin I(A).$$

Denote by $X$ the set of systems which are linked by channels to both $A$ and $B$:

$$X = I(A) \cap I(B)$$

And denote by $Y$ and $Z$ the complements of $X$ in $I(A)$ and $I(B)$:

$$Y = I(A) - X$$

$$Z = I(B) - X.$$

Using arrows to indicate channels, Figure 2 shows all channels not ruled out by the above assumptions and conventions.

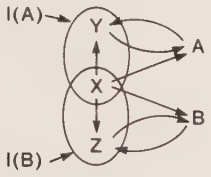


FIGURE 2

There are two cases to consider: either $X$ is empty or it is not. If it is empty then $A \cup I(A)$ and $B \cup I(B)$ are disjoint and entirely noninteracting subsections of the universe $\mathscr{S}$ and no communication between $A$ and $B$ is possible. (This will be a corollary of the general result to follow.) If $X$ is non-empty then some system(s) linked to $A$ is (are) also linked to $B$. Though it is not strictly necessary, it is then *possible* for $X$ to affect both $A$ and $B$ in such a way that their behavior is correlated. This is the basis for communication between $A$ and $B$ without a channel between them.

The technical argument to follow will reveal the conditions necessary for such communication, and the ensuing discussion will discuss the result.

## IV TECHNICAL ARGUMENT

We will start by assuming $Y$ and $Z$ to be empty as in Figure 3. Later it will be easy to generalize the result.

Since $B$ is affected only by $X$ and not by $A$, $p(b \mid x) = p(b \mid xa_i)$ for all values of $a_i$. Converted to entropies this implies that $H_X(B) = H_{X,A}(B)$, and converted to transmissions that $T_X(A:B) = 0$:

$$T_X(A:B) = H_X(B) - H_{X,A}(B) = 0.$$

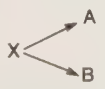


FIGURE 3

By well-known identities[2] in information theory [A3, A4],†

$$T(X:A:B) = T(X:A) + T(X, A:B)$$
$$= T(X:A) + T(X:B) + T_X(A:B) \quad (1)$$

Since $T_X(A:B) = 0$, we get

$$T(X:A:B) = T(X:A) + T(X:B)$$

Also [A3],

$$T(X:A:B) = T(A:B) + T(X:A, B). \quad (2)$$

Combining (1) and (2) and solving for $T(A:B)$,

$$T(A:B) = T(X:A) + T(X:B) - T(X:A, B) \quad (3)$$

With one eye on Figure 3 the terms in this equation can be interpreted as follows:

$T(X:A)$ = amount of information received by $A$ from $X$

$T(X:B)$ = amount of information received by $B$ from $X$

$T(X:A, B)$ = amount of information sent by $X$ to the couple $A, B$.

Other interpretations are possible but this sheds the following light on the situation of communication without a channel: $T(A:B)$ can be positive if the amount of information received by $A$ and $B$ individually exceeds that sent to $A$ and $B$ as a couple. There is no law of "conservation of information," and the amount of information received can exceed that sent, in the above sense, if there is some sort of duplication or overlap in the information sent to the individuals.

$T(A:B)$ = amount of information received—amount of information sent
= amount of overlap, duplication, correlation, etc. in the information sent to the individuals.

If $T(A:B)$ is positive, knowledge of $A$ can be used

†These identities are given in the appendix and reference is made to them here by number.

to reduce uncertainty about $B$. This comes about without a direct channel because from knowledge of $A$ one can reduce uncertainty of $X$ (if $T(X:A)>0$), and from this partial knowledge of $X$ one can in general reduce uncertainty of $B$ (if $T(X:B)>0$). When this two-stage process fails it is because the knowledge of $X$ given by $A$ is irrelevant to those aspects of $X$ which affect $B$, i.e., there is no correlation in the information sent to $A$ and $B$.

This informal argument suggests that $T(A:B)$ is bounded by $T(X:A)$ and $T(X:B)$. To demonstrate this we can invoke the following identity [$A4$]:

$$T(X:B)-T(X:A,B)=-T_B(X:A).$$

Since all transmissions are nonnegative,

$$T(X:B)-T(X:A,B)\leqq 0. \tag{4}$$

Combining (3) and (4), we deduce that

$$T(A:B)\leqq T(X:A)$$

and by symmetry that

$$T(A:B)\leqq T(X:B).$$

Thus

$$T(A:B)\leqq \min\{T(X:A),T(X:B)\}. \tag{5}$$

Consequently we conclude that communication without a channel between $A$ and $B$ is possible only when three conditions are met:

1) The channel $X\rightarrow A$ exists and is active, i.e., $T(X:A)>0$.

2) The channel $X\rightarrow B$ exists and is active, i.e., $T(X:B)>0$.

3) There is overlap or correlation in the information sent by $X$ to $A$ and $B$.

If $X$ is empty, $T(X:W)=0$ by definition. With (5) this verifies the earlier assertion that when $X$ is empty, $T(A:B)=0$.

We may now withdraw the assumption that $Y$ is empty. See Figure 4. Again using standard identities [$A3$, $A4$],

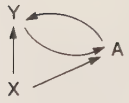

FIGURE 4

$$\begin{aligned}T(X:Y:A:B)&=T(X:A)+T(X,A:B)\\&\quad+T(X,A,B:Y)\\&=T(X:A)+T(X:B)+T_X(A:B)\\&\quad+T(X,A,B:Y)\\&=T(X:A)+T(X:B)\\&\quad+T(X,A,B:Y)\end{aligned} \tag{6}$$

by using $T_X(A:B)=0$ as before. Next [$A3$],

$$\begin{aligned}T(X:Y:A:B)=T(A:B)+T(X:A,B)\\+T(X,A,B:Y).\end{aligned} \tag{7}$$

Combining (6) and (7) we obtain the same result as before:

$$T(A:B)=T(X:A)+T(X:B)-T(X:A,B). \tag{3}$$

Similarly, withdrawing the assumption that $Z$ is empty and thus considering the general condition of Figure 2, the same result holds. It is not surprising, since the conclusion from (5) was that only systems with active channels to both $A$ and $B$ could produce positive $T(A:B)$, and $Y$ and $Z$ each have a channel to only one of the two. The difference between the general case of Figure 2 and the special case of Figure 3 is only that in the general case the effects of $X$ on $A$ and $B$ may in part be mediated by $Y$ and $Z$. The result (3) is true in either case.

The case of Figure 3 may easily be generalized as in Figure 5. The transmission within the

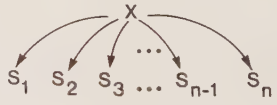

FIGURE 5

receiving variables is best thought of as mutual coordination. The result of generalizing (3) is

$$T(S_1:S_2:\cdots:S_n)=\sum_{i=1}^{n}T(X:S_i)-T(X:S_1,S_2,\ldots,S_n)$$

mutual coordination = amount of information received by individuals − amount of information sent by $X$.

As an illustration of coordination within a group without any internal information exchange, consider a military drill performance; the drill sergeant barks the commands which ideally result in identical movements by the marchers. An orchestra concert provides another example; the orchestra is dependent primarily on the conductor for its synchronized activity.

## V INTERPRETATION AND DISCUSSION

The drill instructor and orchestra leader do not represent unusual cases; it is very usual for a common cause to induce correlated effects: the public news media provide another example, in which the effects—changes in the readers' or listeners' knowledge of the world—are highly correlated, even though they are not visible or overt behaviors. Often it is only because of this type of communication that other communication *using* a channel makes sense, for example this conversational fragment: "Too bad about the air crash." "Yes, horrible." This is only a realistic conversation if both conversants had recently read or heard of a crash (independently) through the news media.

Communication without a channel is used in what is often called feed-forward regulation. Here the regulator or controller makes use of *input* information to the system being controlled. This is used to predict the system's behavior, and the regulator adopts an appropriately matched counter behavior. The result is that the behavior of the system and that of the regulator are highly correlated; by observing one of them one gains a lot of information about behavior of the other.

It may be that some studies of communicatory behavior would be enhanced by considering the type of communication discussed here. For example it seems that investigators of alleged ESP phenomena devote considerable effort to attempts to discover "the channel" which "must" link subjects which seem inexplicably to be in communication in the statistical sense. It may be a fresh insight that such subjects need not be linked by a channel at all and that the cause of the phenomena may lie elsewhere.

Sometimes there does seem to be a qualitative difference between communication with and without a direct channel, and it may be that each type is capable of communicating forms of information that the other is not. For example, if you sit and silently watch a beautiful sunset with a friend there certainly is a type of communication established between you that words cannot produce. Again, certain forms of Buddhism are quite clear in insisting that important truths cannot be conveyed in words but only by the teacher and student sharing certain experiences, and so on.

On the other hand communication without a channel can be a simple substitute for direct communication, as in this scenario: $A$ and $B$ agree to meet outside room 201, where they intend to attend a lecture together. Individually they arrive at room 201 to find a notice stating that the lecture has been moved to room 430. Each goes to room 430, and they meet there. The notice has induced communication between them just as surely as if one had said explicitly to the other, "I'll meet you outside room 430."

Once it becomes clear that the sharing of common experiences or influences creates communication between those who are influenced, a new perspective on communication appears. Those who grow up together in a culture are subjected to decades of common influences, in part by the laws of the physical world and in part by the culture. We have common mental models of the way things work, of language, social customs, appropriate behavior, and so on. It is the correlation between these models, shaped by common experience, that represents communication without a direct channel between two people who have never met. The fact that two strangers in the culture can carry on a coherent conversation illustrates the importance of this *prior* communication. For contrast imagine a New Yorker trying to communicate with an Australian Aborigine or a Martian.

The fact that correlation of mental models can substitute for explicit communication over a channel is a further argument that these both represent communication, but in different forms. In tightly constrained circumstances very little channel-communication is needed. Witness these brief scenarios:

1) I walk into a small restaurant at 7 a.m. A waitress comes up and looks at me silently. I say "Two over easy." She leaves. I know that she understands what I want for breakfast.

2) I walk up to a ticket booth, put down $5. The agent hands me $1.50 and a ticket. I leave. No words were needed at all.

From the perspective of this article, then, it seems apparent that a great deal of everyday communication does not involve a direct channel between the communicators. No doubt both channel—and *non*-channel—communication are of great importance, but although it would be very difficult to verify it quantitatively, it may be that communication without a channel is actually the dominant and more important form.

REFERENCES

1. C. E. Shannon, "The mathematical theory of communication." *Bell Syst. Tech. Journal*, **27,** July, 1948, pp. 379 423; October, 1948, pp. 623–656. Reprinted in book form: Urbana, University of Illinois Press, 1963.
2. W. Ross Ashby, "Two tables of identities governing information flows within large systems." *Commun. Amer. Soc. Cyb.*, **1,** No. 2, 1969, pp. 2–8.
3. R. Conant, "Information flows in complex systems." *Int. J. General Systems*, **1,** No. 1, 1974, pp. 9–18.

## VII APPENDIX: Identities in Information Theory

These can be found in Ashby's article.[2]

$$[A1]\quad H(S_1, S_2, \ldots, S_n) = H(S_1) + H_{S_1}(S_2) + H_{S_1, S_2}(S_3) + \cdots + H_{S_1, (S_n)S_2, \ldots, S_{n-1}}$$

$$[A2]\quad T(S_1 : S_2 : \cdots : S_n) = \sum_{i=1}^{n} H(S_i) - H(S_1, S_2, \ldots, S_n)$$

$$[A3]\quad T(S_1 : S_2 : \cdots : S_n) = T(S_1 : S_2) + T(S_1, S_2 : S_3) + T(S_1, S_2, S_3 : S_4) + \cdots + T(S_1, S_2, \ldots, S_{n-1} : S_n)$$

$$[A4]\quad T(S_1 : S_2, S_3, \ldots, S_n) = T(S_1 : S_2) + T_{S_2}(S_1 : S_3) + T_{S_2, S_3}(S_1 : S_4) + \cdots + T_{S_2, S_3, \ldots, S_{n-1}}(S_1 : S_n)$$

*For biography and photograph of the author, please see Vol. 1 No. 1, Jan. 1974, p. 74 of this journal.*

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# THE CONCEPT OF STRUCTURAL ENTROPY AND ITS APPLICATIONS TO GENERAL SYSTEMS†

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The concept of relativistic structural entropy is used to describe capture phenomena and hierarchical dynamics in general systems. After a background on relativistic information and the relativistic description of general systems, as previously introduced by the author, the dynamics of hierarchical systems which modify their hierarchy are considered; and the problem is completely solved for a quantum model composed of three elements. Conditions for capture and hierarchy are given, as well as conditions for isolation. It is shown that this approach may be a way to attack the morphogenesis problem considered as a problem of growing of forms, and the connection with the catastrophe theory by Thom emerges. Some examples are given to illustrate how one can identify the variables of the general model in practical problems.

INDEX TERMS Relativistic description of general systems, relativistic information, structural entropy, effective entropy, special relativity, structural dynamics, hierarchical systems, self-organizing systems.

## 1 INTRODUCTION

### *1.1 The Main Motivations*

The concept of system is so pervasive in science that, apparently, we can deal with it without preliminary definition. But in fact, by looking more closely, defining general systems could raise difficulties similar to those of classes or sets in mathematics (which came the first, the chicken or the egg?) and to avoid this, we shall straightforwardly state some considerations which could be the main motivations of a general systems theory.

i) We are investigating systems of higher and higher dimensions: these systems involve components which are physically different, and we need a unified approach to study their interaction. For instance, the world society has an economic component, a societal component, a political component, and energizing component, etc.... and we have to exhibit their common features to analyze their joint evolution.

ii) All dynamics on a large time scale can be theoretically described with enough accuracy by partial differential equations. But while this approach is highly efficient for local dynamics on a relatively small interval of space and time, it seems to be rather questionable when applied to global dynamics involving space and the horizon as a whole. This doubt stems partly from the burdensome computations and partly from the complexity of the models that the classical study provides, and in this way a recent six hundred-variable model for the Canadian economy is quite instructive. But while pure topological approaches do not yield enough quantitative results, differential equations involve too many numerical results which put an obstacle in the way of analyzing the structural dynamics of general systems.

iii) Experiment has shown that hierarchical structures play a crucial role in self-organizing systems, and in effect, a general model for general systems should aim to easily (with simplicity and efficiency) describe hierarchy in order to provide a way to suitably investigate hierarchical dynamics.

iv) A general model for general systems has been recently proposed by Thom[1] who describes and enumerates the basic forms of our universe. This highly theoretical investigation suggests a few disturbing questions to the physicist: What is the physical meaning of the potential function? How can we identify the potential function of a given system? How can we physically explain that we have only seven basic forms?

The purpose of the present paper, which is more particularly directed to the dynamics of hierarchies, is to provide a formulation which

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could be a way to attack these questions. The article is composed of two parts: in the first one, we give a brief background on relativistic information and our relativistic construction for general systems; in the second one, the concept of structural entropy is applied to the structural dynamics of hierarchical systems and identification with usual systems are given.

### *1.2 The Closing-Opening duality*

Until recently a few mathematicians expected to confine the universe in closed mathematical systems which would offer all guarantees concerning scientific objectivity. We now know that a wonderful answer has been brought by Gödel in 1931 with his famous theorem on the incompleteness of formal systems (see for instance Davis[2]) that we can consider as the mathematical version of the closing-opening duality in general systems: that is to say, a system should be open to be observed, and it should be closed to be defined. But as Morgenstern said,[3] it is now fortunately and definitively obvious that preliminary descriptions (or heuristic reasonings) and rigorous theories are mutually dependent and they are both necessary for the advancement of sciences.

In this way, we invite the reader to consider the following as a preliminary description by means of which it will be possible to formulate a rigorous theory later.

LIST OF SYMBOLS AND NOTATIONS

| | |
|---|---|
| $U$ | universe |
| $(S/R)$ | system $S$ given the observer $R$ |
| $(\bar{S}/R)$ | environment of $S$ given the observer $R$ |
| $(S/R/R')$ | system $S$ observed by $R'$ through $R$ |
| $\alpha$ | random experiment |
| $H(\alpha)$ | Shannon entropy, or Sh-entropy of $\alpha$ |
| $H_i(S/R)$ | internal entropy of $S$ given $R$ |
| $H_0(S/R)$ | external entropy of $S$ given $R$ |
| $v(S/R)$ | objective of $S$ given $R$ |
| $w(S/R)$ | transformation potential of $S$ given $R$ |
| $u(R/R')$ | organizability of $R$ given $R'$ |
| $c$ | organizability of $U$ given $U$ |
| $\sigma$ | metric state of $(S/R)$ |
| $I_i(S/R/R')$ | internal relativistic information provided by $R$ to $R'$ about $S$ |
| $I_0(S/R/R')$ | external relativistic information provided by $R$ to $R'$ about $S$ |
| $I(S/R/R')$ | metric of the relativistic information provided by $R$ to $R'$ about $S$ |
| $E(S/R/R')$ | relativistic structural entropy of $S$ given $R'$ via $R$ |
| $E(S/S/R')$ | direct structural entropy of $S$ given $R'$ |
| $\rho(S/R)$ | coefficient $[1-u^2(S/R)/c^2]^{-\frac{1}{2}}$ |
| $K(S/R)$ | coefficient $\rho(S/R)u/S/R)/c$ |

## PART I

## A BRIEF INTRODUCTION TO RELATIVISTIC SYSTEMS

### 2 GENERAL FRAMEWORK

Our purpose is to obtain a unified approach to systems so different as biological systems, societal systems, economic systems, linguistic systems, etc.... So, we shall consider a given universe, say a universe of discourse, which is defined by common physical characteristics from the viewpoint of a given observer; and a system will be a set in this universe. Formally we have:

DEFINITION 1 Let $U=\{e_i\}$ be a collection of objects denoted generically by $e_i$. *$U$ is defined as the universe.* A *system $S$* is any part of $U$. □

*Example* $U$ may be composed of biological cells. $U$ may be a linguistic universe composed of symbols, for instance integer numbers, and concrete objects, for instance a table or a car.

This being so, a given system may be considered from different viewpoints by different observers. A newspaper, for instance, which is a subset of the linguistic universe is considered differently by an idle workman and a businessman. We shall summarize this basic feature in the following statement.

AXIOM 1 *A system $S$ is defined only with respect to a given observer $R$ who observes it: say $(S/R)$.* □

The elements $e_i$ which compose the system define its *inside* by contrast with the other ones which are its *outside*. The elements of the inside are arranged in a certain organization which we shall call by the generic term of *internal structure* $\phi$ of $(S/R)$. Since the system is defined given an observer $R$, it will be so for its internal structure $\phi$ which ranges over a configuration space $F$. The system is subject to inputs from its outside, or more exactly from its *environment* $\bar{S}$ which may be a part of this outside only, and we shall refer to these inputs as *information* I which is a generic term. We state:

AXIOM 2 *A system $(S/R)$ receives information from its environment $(\bar{S}/R)$; it uses this information to modify its internal structure and to act on $(\bar{S}/R)$ in order to achieve a certain objective.* □

AXIOM 3 *Let dI denote the amount of information that the system receives from its environment in an infinitely small evolution. Let* $n(\phi): F \to R$, $\phi \mapsto n(\phi)$, *denote a function which we call the negentropy of S, and which represents the maximum amount of knowledge that the system has stored about its environment. The dynamics of the system are governed by the following principle.*

*Evolution principle* Assume that the system evolves from a state $\phi_1$ to another one $\phi_2$; then this evolution is such that the difference

$$E_2 - E_1 \triangleq \int_{\phi_1}^{\phi_2} \frac{dI}{n(\phi)} \tag{1}$$

is maximum.

DEFINITION 2 The function $E$ so defined by Eq. (1) is called the *structural entropy* (St-entropy) of the system.

Remark that $E$ is nothing else but the potential function introduced by Thom, but here we give a precise definition which is suggested by thermodynamics. The integral of Eq. (1) is not clearly defined for the moment as the space $F$ itself is not rigorously defined; this will be done shortly according to the feedback principle of Subsection 1.2.

AXIOM 4 *The universe is blind in the sense that it cannot observe its own evolution.* □

From a philosophical viewpoint, this axiom introduces a bound on the amount of knowledge that we can acquire. For instance, in a biological framework, we can postulate that we will never be able to determine how did the mankind come. In a psychological framework somebody can hardly have a fair opinion about himself. From a mathematical viewpoint this axiom provides the exact definition of the metric of the system (see Definition 9).

## 3 DESCRIPTION OF GENERAL SYSTEMS

### *3.1 The Universe*

Since the environment $\bar{S}$ is the only part of $U$ with which $S$ can exchange information, then from the viewpoint of $S$, the universe is reduced to $\bar{S}$, say

$$U(S/S) = \bar{S}.$$

As we are an observer, and thus a system, we cannot directly observe the universe as a whole, and in effect we cannot rigorously define it. To avoid this difficulty, we shall consider the universe as the union of the systems which compose it.

DEFINITION 3 Let $\{R_i, i \in IcN\}$ denote a set of given observers $R_i$. Let $\{S_j, j \in JcN\}$ denote the set of the systems $S_n$ which can be observed by the observers $R_i$'s, in the sense that if $R_i$ comes into the environment $\bar{S}_j$ of $S_j$, then there will be an exchange of information between $R_i$ and $S_j$. The Universe $U$ is defined as the union

$$U = \bigcup_j \bar{S}_j. \quad \square \tag{2}$$

### *3.2 Internal Entropy* $H_i(S/R)$

A system is, in fact, defined by its inside and its outside; an observer $R$ will have a good knowledge of the system under observation if and only if he has a good knowledge of both the inside and the outside of the system, since the dynamics of the latter is defined by the exchange of information between this inside and this outside.

DEFINITION 4 The amount of knowledge that the observer $R$ has about the internal structure of the system $(S/R)$ is denoted by $H_i(S/R)$ and is called by the generic term *internal entropy of S given R.* □

The definition of this knowledge is itself a structural parameter which is varying from an observer $R$ to another one $R'$ and from a system $S$ to another one $S'$. This knowledge can be defined as a set of differential equations, a hierarchical graph, or a Shannon entropy as well (Sh-entropy in the following). In such a case one considers the determination of the internal structure of the system as the result of a random experiment, and $H_i(S/R)$ is identified with the Sh-entropy of the latter.

### *3.3 External Entropy* $H_0(S/R)$

DEFINITION 5 The *external entropy* $H_0(S/R)$ *of a system S given an observer R,* represents the amount of knowledge that $S$ has about the internal structure of its environment $\bar{S}$, but as measured by $R$. □

In fact, one has

$$H_0(S/S) \triangleq H_i(\bar{S}/S)$$

and the estimate $H_0(S/R)$ of $H_0(S/S)$ by $R$ is

$$H_0(S/R) = H_i(\bar{S}/S/R).$$

Remark that one has

$$H_i(\bar{S}/S/R) \neq H_i(\bar{S}/R)$$

since $H_i(\bar{S}/R)$ defines the direct observation of $\bar{S}$ by $R$, while $H_i(\bar{S}/S/R)$ describes the two-level observation of $\bar{S}$ by $R$ through $S$. As with the internal entropy, it may be helpful to consider $H_0(S/R)$ as a Sh-entropy.

### 3.4 *Objective $v(S/R)$*

DEFINITION 6 Every system has an objective to perform, and this objective is defined by the variable $v(S/R)$. □

This objective may be quantitative, for instance it may be the minimization of a given functional; it may also be a qualitative or topological objective, as to remain inside a given class. An important class of systems is that which has a constant objective, i.e. $dv(S/S)=0$; for instance, the objective of a living being would be to die. Remark that, when we write $v(S/S)=$constant, we make an assumption, since we cannot measure $v(S/S)$, and all we have at hand is the observed $v(S/R)$ which is varying from an observer $R$ to another one $R'$. So $R$ can make the assumption $dv(S/R)=0$ while $R'$ can assume $dv(S/R')\neq 0$, and this is exactly the relativistic feature which appears to be basic in general systems.

### 3.5 *Transformation Potential $w(S/R)$*

DEFINITION 7 The *transformation potential $w(S/R)$ of a system $S$ given an observer $R$* defines the efficiency of the transfer of the information inside the system. □

This efficiency is directly related to the variations of the internal structure of the system, therefore the term of transformation potential. For instance, if $S$ is composed of two subsystems $S_1$ and $S_2$ isolated from each other, then there is no transfer of information from $S_1$ and $S_2$, and the internal structure of $S$ will be invarying. Furthermore, the higher the information transfer is, the more the system is subject to internal change. It follows that, if we consider the smaller observable system, say a cell, we can say that its transformation potential is zero since there is no information transfer in its inside.

## 4 RELATIVISTIC FEATURES OF GENERAL SYSTEMS

### 4.1 *Internal Entropy*

The basic remark is the following. We cannot rigorously assert that a given system $S$ has the same internal entropy $H_i(S/R_i)$ for every observer $R_i$. □

This can be supported as follows

In the Shannon framework, assume that the determination of the internal structure of $S$ is a random experiment, i.e. a complete system of events $\alpha=\{A_i; i=1,\ldots,n\}$ which yields outcomes $A_i$ with the probability $p(A_i)$; and identify the internal entropy of $\alpha$ with its usual Sh-entropy, i.e.

$$H(\alpha) = -\sum_i p(A_i)\ln p(A_i).$$

The Sh-entropy so appears as an absolute entropy which is independent of the observer $R$. In fact, the uncertainty about $\alpha$, for a given observer $R$, can be decomposed into two parts: the uncertainty $H(\alpha)$, of random feature, which is related to the prediction of the element $A_i$ which will occur; and a uncertainty $h(\alpha/R)$, say the *observer uncertainty*, which can be defined as follows.[5] When $\alpha$ yields the outcome $A_i$, $R$ observes $A_i$ with an uncertainty $\eta(A_i/R)$, and $h(\alpha/R)$ is taken in the form of the weighting sum

$$h(\alpha/R) = \sum_i p(A_i)\eta(A_i/R).$$

So, from a general point of view, the complete uncertainty, i.e. the *effective uncertainty* $H_e(\alpha/R)$ of the observer $R$ regarding $\alpha$ is a certain function, say

$$H_e(\alpha/R) = F[H(\alpha),\ h(\alpha/R)],$$

and it is easily understandable that, since $h(\alpha/R)$

is varying from an observer $R_1$ to another one $R_2$, the equation

$$F[H(\alpha), h(\alpha/R_1)] = F[H(\alpha), h(\alpha/R_2)]$$

generally will not hold for every pair $(R_1, R_2)$. □

We consider the dependence of $h(\alpha/R)$ on $R$ as an experimental fact; the following example may serve as an illustration.

People who live in a certain city $C$ always speak the truth, while the residents of a bordering town $S$ are always telling lies. An observer $R$ knows that he is in one of these cities, but he does not know in which one. $R$ wants to determine the city in which he stands, and to this end he puts questions to the first person that he meets in the street. People answer by "*yes*" or "*no*"; inhabitants of $C$ can be passing by $S$, or conversely, people who live in $S$ can go to $C$. The problem for $R$ is to determine the smallest number of questions which he needs to pose, to define these questions, and to determine the city he is in.

It is well known that the answer to the question "*Do you live in this town?*" solves the problem; but it is an experimental fact that an observer $R_1$ will be effectively capable of finding the solution, while another one $R_2$ will not. The amount of information which is contained in the answer is not the same for $R_1$ and $R_2$. Linguists refer to such a situation by saying that the semantics of the message is not the same for $R_1$ and $R_2$.

### 4.2 *External Entropy*

As with the internal entropy, the external entropy is not an absolute concept which can be defined independently of any reference, and we cannot rigorously state that two different systems $S_1$ and $S_2$ have the same external entropy.

This is basically due to the fact that the external entropy of $S$ is the internal entropy of its environment $\bar{S}$, so that, by virtue of Subsection 4.1, it will depend upon the observer.

Assume that $S_1$ and $S_2$ compose a system $S$ as shown in Figure 1. The entropy $H_0(S_1/S_1)$ or $H_0(S_2/S_2)$ is determined by an observer $R$ which, to this end, exchanges information with $S_1$ or $S_2$ as shown in Figure 2 where $S'$ denotes the new system composed of $S_1, S_2$ and $R$.

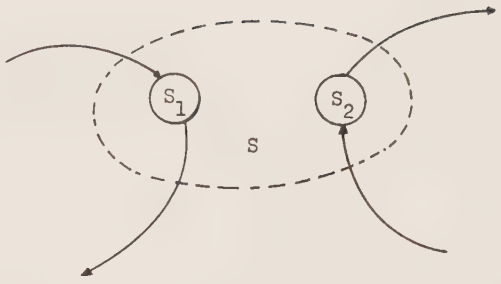



FIGURE 1 System $S$ composed of $S_1$ and $S_2$ isolated each other.

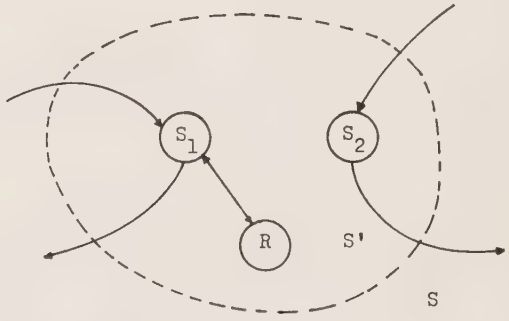



FIGURE 2 Determination of $H_0(S_1/R)$ by $R$.

Now, since $R$ neither receives information from $\bar{S}'$, nor acts on it, one has

$$H_0(S/S) = H_0(S'/S') \triangleq \hat{H}.$$

This being so, the results of the experiments by $R$ are given by the equations

$$H_0(S_1/R) = \hat{H} + H_i(R/S_1)$$

$$H_0(S_2/R) = \hat{H} + H_i(R/S_2),$$

in such a manner that the identity

$$H_0(S_1/R) = H_0(S_2/R)$$

holds only when

$$H_i(R/S_1) = H_i(R/S_2),$$

that is to say, when $S_1$ and $S_2$ have the same degree of knowledge about $R$. It is clear that this equality generally will not hold for every pair $(S_1, S_2)$.

*4.5 Specific Properties of the Universe*

We state:

PROPOSITION 1 *For the universe U, the following equation holds:*

$$H_i(U/U)=cH_0(U/U) \tag{3}$$

where $c$ denotes a constant which depends upon the measurement units. □

*Proof* (i) Consider Figure 2 which represents the observation process of $S$ by $R$, and assume that $R$ does not belong to $\bar{S}$: $R\notin\bar{S}$. One has

$$H_0(S/R)=H_i(\bar{S}/S/R), R\notin\bar{S}. \tag{4}$$

ii) Assume now that $R$ comes into $\bar{S}$, and denote by $\bar{S}-R$ the complement of $R$ *w.r.t.* $\bar{S}$: $\bar{S}=RU\bar{S}-R$. We then have

$$H_0(S/R)=H_i(\bar{S}-R/S/R)+H_i(R/S/R), R\in\bar{S}. \tag{5}$$

iii) We now assume that $U$ is observing itself. As $U$ exchanges information with itself, Eq. (4) does not hold, and Eq. (5) yields

$$H_0(U/U)=H_i(U-U/U/U)+H_i(U/U/U). \tag{6}$$

The definition of internal entropy provides

$$H_i(\bar{U}-U/U/U)=H_i(\bar{U}-U/U)$$

and

$$H_i(U/U/U)=H_i(U/U).$$

Furthermore, as the larger system which exchanges information with $U$ is $U$ itself, one has $\bar{U}=U$. The system $\bar{U}-U$ is then the empty set; there is no uncertainty on its structure, and we have $H_i(\bar{U}-U/U)=0$. We finally derive from Eq. (6), the condition

$$H_0(U/U)=H_i(U/U)$$

which is generalized in the form of Eq. (3). This completes the proof.

## 5 RELATIVISTIC MODEL FOR GENERAL SYSTEMS

*5.1 Organizability of a System*

The particular importance of the external entropy suggests the introduction of the following definition.

DEFINITION 8 The *vector organizability* $g(S/R)=\{g_1,g_2,g_3\}_{S/R}$ of a system $S$ given an observer $R$, is the vector whose components are defined by the equations

$$g_1(S/R)\triangleq dH_i(S/R)dH_0(S/R) \tag{7a}$$

$$g_2(S/R)\triangleq dv(S/R)dH_0(S/R) \tag{7b}$$

$$g_3(S/R)\triangleq dw(S/R)dH_0(S/R) \tag{7c}$$

and the *scalar valued organizability* $\gamma(S/R)$ of $S$ given $R$ is defined by the equation

$$\gamma^2(S/R)\triangleq g_1^2(S/R)+g_2^2(S/R)+g_3^2(S/R). \quad \square \tag{8}$$

*Notation* The special case in which $g_2\equiv g_3\equiv 0$ while $g_1\neq 0$ is of importance, and we shall refer to it as $g(S/R)=\{u(S/R), 0, 0\}$ with $u(S/R)\equiv g_1(S/R)$.

The similarity of the organizability with the velocity in mechanics is apparent. In fact it is $H_0(S/R)$ which is the independent variable here, and $g(S/R)$ measures how much $(S/R)$ modifies its internal state variables depending upon its knowledge about $(\bar{S}/S/R)$.

Remark that by virtue of Eq. (3), $c$ is the organizability $c=dH_i(U/U)/dH_0(U/U)$ of the universe $U$ given $U$.

*5.2 Relativistic Model*

In the preceding sections we have emphasized the relativistic features of general systems: clearly, a system $S$ is defined only with respect to a given observer $R$ who examines $S$ with his own viewpoint. We have exhibited a universe organizability $c$ which recalls the light velocity in physics, and on the whole it is apparent that the properties of the external entropy $H_0(S/R)$ are similar to those of time in relativistic physics.[1] So we straightforwardly use Einstein's formulation and we state:

DEFINITION 9 A system $S$, given an observer $R$, is represented by the generic point, with coordinates $(H_0(S/R), H_i(S/R), v(S/R), w(S/R)$, in a Riemannian space endowed with the geodesic

$$d\sigma^2(S/R)=c^2dH_0^2(S/R)-dH_i^2(S/R)-dv^2(S/R)-dw^2(S/R),$$

and the observer will measure the evolutions of $S$ when this latter evolves from the state $\phi_1$ to the state $\phi_2$, in the form of the integral

$$\sigma(\phi_2/R)-\sigma(\phi_1/R)=\int_{\phi_1/R}^{\phi_2/R} d\sigma(S/R). \quad \square \qquad (9)$$

Remark that $d\sigma^2(U/U)=0$: this is Axiom 4.

*Lorentz Special Transformations.* Define

$$H_i \triangleq H_i(S/R) \qquad H_0 \triangleq H_0(S/R) \qquad (10a)$$

$$H_i' \triangleq H_i(S/R') \qquad H_0' \triangleq H_0(S/R') \qquad (10b)$$

$$v \triangleq v(S/R) \qquad w \triangleq w(S/R) \qquad (10c)$$

$$v' \triangleq v(S/R') \qquad w' \triangleq w(S/R) \qquad (10d)$$

Let $u$ denote a real-valued constant s.t. $u<c$, and define

$$\rho \triangleq \left(1-\frac{u^2}{c^2}\right)^{-1/2}. \qquad (11)$$

The special transformation of Lorentz is defined by the equations

$$H_i'=\rho(H_i+uH_0) \qquad (12a)$$

$$v'=v \qquad (12b)$$

$$w'=w \qquad (12c)$$

$$H_0'=\rho\left(H_0+\frac{u}{c^2}H_i\right) \qquad (12d)$$

and conversely

$$H_i=\rho(H_i'-uH_0') \qquad (12a)$$

$$v=v' \qquad (12b)$$

$$w=w' \qquad (12c)$$

$$H_0=\left(H_0'-\frac{u}{c^2}H_i'\right). \qquad (12d)$$

### 5.3 *Physical Meaning of the Constant u*

We state the following result.

PROPOSITION 2 *Consider a system $(S/R)$ which evolves with a constant vector organizability $\{u, o, o\}$. Then the internal entropy $H_i(S/S)$ is constant.* $\square$

PROPOSITION 3 *The constant $u$ of the special transformation of Lorentz is such that*

$$u \triangleq u(R/R')=dH_i(R/R')/dH_0(R/R'). \quad \square \qquad (13)$$

For the proof, see Jumarie.[6]

### 5.4 *Meaning of the Lorentz transformation*

In relativistic physics, the Lorentz transformation involves two co-ordinate systems $(0, x, y, z)$ and $(0', x', y', z')$. The first one moves *w.r.t.* the second one with a constant velocity $\dot{u}=dx/dt$ in such a manner that the axes $0x$ and $0'x'$ remain mingled while $0y$ and $0'y'$ on the one hand, and $0z$ and $0'z'$ on the other hand, are respectively parallel: $0y//0'y'$, $0z//0'z'$. In this framework, there is a complete identification between the observer and the coordinate systems, say $(0, x, y, z)\equiv R$; $(0', x', y', z')\equiv R'$, and the Lorentz transformation describes how $R$ and $R'$ observe the same physical event $ds^2$ (see for instance Eddington[7]).

We now consider the Lorentz equation

$$x'=\left(1-\frac{\tilde{u}^2}{V^2}\right)^{-1/2}(x+\tilde{u}t)$$

where $V$ denotes the light velocity and $t$ denotes the time. Theoretically, an observer who is in the co-ordinate system $(0', x', y', z')$ can estimate the observed coordinate $x'$ provided that he has at hand the values of $V$, $x$ and $t$. This can be pictured by stating that $(0', x', y', z')$ *observe the physical event $ds^2$ through* $(0, x, y, z)$.

The model is quite similar for general systems. In fact, it is not at all certain that time should have to be the basic reference in the dynamics of these latters, and this seems to have been confirmed in biology for instance (see Pinel[8,9]). A system modifies its internal structure, its objective and its potential depending upon the amount of knowledge that it acquires about its environment, so that the very parameters of any dynamical system appear to be the external entropy of this latter rather than time itself. It is the remark supporting the determination of the metric of Definition 9 which describes the general dynamics of general systems. This being so, if the reference $R$ evolves *w.r.t.* $R'$ with the constant

organizability $u(R/R')$ expressed by Eq. (13), then the Lorentz special transformation will hold.

There now remains the task of interpreting these equations. As in relativistic physics, $R$ and $R'$ are observing the same system, and moreover they observe it in the same metric

$$d\sigma^2(S/R) = d\sigma^2(S/R').$$

As above, the Lorentz transformation defines the two-level observation process $(S/R/R')$ pictured in Figure 3 in which the *indirect observer* $R'$ *observes* $S$ *through the direct observer* $R$.

FIGURE 3 Lorentz transformation process $(S/R/R')$.

## 6. RELATIVISTIC INFORMATION

The "communication" theory initiated by Shannon[10] is really a theory of communication in the sense that it involves only one of the two characteristics of information, which are the syntax and the semantics. This approach has been generalized mainly from a mathematical viewpoint.[11, 12, 13, 14] However, we believe with Thom[1] and others[15] that information involves subjective factors related to the observer, and an information theory should deal explicitly with this relativistic feature.

In fact, every information process involves the source, or the system $S$ which pours this information, the information $I$ itself, and the observer $R$ who receives it, in such a manner that any information theory should deal with the triplet $(S, I, R)$ which would simultaneously lead to a system theory and an information theory mutually dependent.

In the following, we shall derive a relativistic information model as a by-product of our approach to general systems.

### 6.1 *One-level Relativistic Information*

DEFINITION 10 Consider an indirect observer $R'$ who observes a system $S$ through a direct observer whose vector organizability is $\{u(R/R'), 0, 0\}$.

i) The *internal relativistic information* $I_i(S/R/R')$ provided by $R'$ to $R$ on the internal structure of $S$ is defined by the equation

$$I_i(S/R/R') \triangleq H_i(S/R') - \rho(R/R')H_i(S/R) = \rho(R/R')u(R/R')H_0(S/R). \tag{14}$$

ii) The *external relativistic information* $I_0(S/R/R')$ provided by $R$ to $R'$ on the external structure of $S$ is given by the equation

$$I_0(S/R/R') \triangleq H_0(S/R') - \rho(R/R')H_0(S/R) = \frac{1}{c^2}\rho(R/R')u(R/R')H_i(S/R). \tag{15}$$

iii) The metric $dI^2(S/R/R')$ of the relativistic information is defined as

$$dI^2(S/R/R') \triangleq dI_i^2(S/R/R') - c^2 dI_0^2(S/R/R') = \left(\frac{\rho u}{c}\right)_{R/R'} d\sigma(S/R). \quad \square \tag{16}$$

*Motivation* The definition can be supported as follows. Consider the Lorentz equation (12a) for instance, and assume that we are in the setting of random experiments. So, $H_i(S/R')$ and $H_i(S/R)$ are the uncertainties of $R'$ and $R$ on the internal structure of $S$ respectively, and according to Eq. (12a), the quantity

$$H_i(S/R') - \rho(R/R')H_0(S/R)$$

represents the decreasing of the uncertainty $H_i(S/R')$ when $R'$ observes $S$ through $R$. This is thus the amount of information provided by $R$ to $R'$ on the internal structure of $S$, therefore Eq. (14).

Equation (16) is derived from the fact that $\sigma(S/R)$ itself represents the information that $R$ has about $S$, while $\rho u/c)_{R/R'}$ is the term involved by the observation process of $R'$.

### 6.2 *Multi-level Information*

We now consider an observer $R''$ who observes the system $S$ through the cascaded combination $(S/R/R'/R'')$ as shown in Figure 4.

FIGURE 4 Two-level observation process.

In addition to notations (11a–11d) we put

$$H_i'' \triangleq H_i(S/R'') \qquad H_0'' \triangleq H_0(S/R'') \tag{17a}$$

$$u' \triangleq u(R'/R'') \qquad \rho' \triangleq [1-(u'^2/c^2)]^{-1/2} \tag{17b}$$

We then have the following definition.

DEFINITION 11 Consider the two-level observation process in which the indirect observer $R''$ observes the system $S$ through the cascaded combination $(S/R/R'/R'')$. Assume further that the organisabilities of $R$ and $R'$ are respectively $\{u(R/R'),0,0\}$ and $\{u(R'/R''),0,0\}$.

i) The *internal relativistic information* $I_i(S/R/R'/R'')$ provided by $(R/R')$ to $R''$ on the internal structure of $S$, is defined by the equation

$$I_i(S/R/R'/R'') \triangleq H_i'' - \rho\rho'\left(1+\frac{uu'}{c^2}\right)H_i$$
$$= \rho\rho'(u+u')H_0 . \tag{18}$$

ii) The *external relativistic information* $I_0(S/R/R'/R'')$ provided by $(R/R')$ to $R''$ on the external structure of $S$ is defined by the equation

$$I_0(S/R/R'/R'') \triangleq H_0'' - \rho\rho'\left(1+\frac{uu'}{c^2}\right)H_0$$
$$= \rho\rho'\frac{uu}{c^2}H_i . \tag{19}$$

iii) The geodesic $dI^2(S/R/R'/R'')$ is

$$dI^2(S/R/R'/R'') = \rho\rho'\frac{uu'}{c^2}\,d\sigma^2(S/R) . \tag{20}$$

Definition 11 is a straightforward generalization of Definition 10. □

## 7 RELATIVISTIC STRUCTURAL ENTROPY

### 7.1 *Definition*

In Section 2 related to the main axioms, we have introduced a structural entropy concept, which needs a more accurate definition in our relativistic approach. We state:

DEFINITION 12 Consider an indirect observer $R'$ who observes a system $S$ in the Lorentz observation process $(S/R/R')$. The *relativistic structural entropy* $E(S/R/R')$ of $S$ given $(R/R')$, *R st-entropy* in the following, is defined by the equation

$$dE(S/R/R') \triangleq \frac{1}{c}\rho(R/R')u(R/R')\frac{d\sigma(S/R)}{\sigma S(/R)} \tag{21}$$

$$= \frac{dI(S/R/R')}{\sigma(S/R)} . \tag{22}$$

The *direct relativistic structural entropy* of $S$ given $R'$, *D rst-entropy* in the following, is defined as to be the quantity $E(S/S/R')$ obtained by substituting $S$ for $R$ in Eq. (21). □

The motivation of this definition may be summarized as follows. While $dI(S/R/R')$ is meaningful with regard to the term $dI$ of Eq. (1), the identification of $\sigma(S/R)$ with $n$ is less transparent. In fact Eq. (21) shows that $dI(S/R/R')$ and $d\sigma(S/R)$ have the same physical dimension, while $(\rho u/c)$ is similar to a gain coefficient, in such a manner that $(\rho u/c)d\sigma$ represents the part of $d\sigma$ that $R'$ gets effectively. Therefore the negentropy equivalence of $\sigma(S/R)$.

### 7.2 *Some Results*

*Notation* In the following, we shall set

$$K(S/R) \triangleq \frac{1}{c}\rho(S/R)u(S/R) . \tag{23}$$

With this notation the *Rst*-entropy has the form

$$dE(S/R/R') = K(R/R')d\sigma(S/R)/\sigma(S/R) . \tag{24}$$

This being so, we state

PROPOSITION 4 *Assume that the system* $(S/S)$ *has a constant objective* $v(S/S)$ *and a constant transformation potential* $w(S/S)$*; then the following equation holds:*

$$dE(S/S/R') = K(S/R')dH_0(S/S)/H_0(S/S) . \quad \square \tag{25}$$

*Indeed, according to Proposition* 2, $H_i(S/S)$ is constant. *If we set* $H_i(S/S)=0$, *we then have*

$$d\sigma^2(S/S) = c^2 dH_0^2(S/S),$$

*therefore the result.*

PROPOSITION 5 *Assume that the system* $(S/R')$ *has a constant objective* $v(S/R')$ *and a constant transformation potential* $w(S/R')$. *Assume further that* $g(S/R')\equiv\{u(S/R'),0,0\}$ *is constant. Then the following equation holds:*

$$dE(S/S/R')=K(S,R')dH_0(S/R')/H_0(S/R'). \quad \square \tag{26}$$

*Indeed, as* $H_i(S/S)$ *is constant, Eq.* (12*d*) *of the Lorentz transformation yields,*

$$dH_0(S/R')=\rho(S/R')dH_0(S/S),$$

*therefore the result.*

*We then have*

$$E(S/S/R')=\ln H_0(S/R')^{K(S/R')} \tag{27}$$

*provided that the origin is* $H_0(S/R')=1$.

## 8 STRUCTURAL DYNAMICS OF GENERAL SYSTEMS

### *8.1 General Statement*

We state the evolution principle in the following form.

*Relativistic Evolution principle.* A system $S$ given an observer $R'$ evolves in such a manner that it maximizes its direct relativistic structural entropy $E(S/S/R')$. $\square$

As is evident, the dynamics of $S$ should be absolute and independent of $R'$, say $E(S/S)$. But such a dynamics would be unobservable for $R'$, and in effect would have no interest, since the system is described from the point of view of $R'$. So the most possible direct observation of $S$ by $R'$ is that which provides $E(S/S/R')$, therefore the statement.

### *8.2 Composition Law for the Relavistic Structural Entropy*

We can now state the following result.

PROPOSITION 6 (*i*) *Let* $\{S_i: i=1,2,\ldots,n\}$ *denote a set of systems which compose an overall system* $S$;

$$S\triangleq\{S_1,S_2,\ldots,S_n\}$$

*ii) Let* $\{\{R_i\}: i=1,\ldots,n\}$ *denote a set of indirect observers,*

$$\{R_i\}\triangleq R_i^1/R_i^2/\ldots/R_i^{n_i},$$

*by means of which the observer* $R'$ *observes* $S_i$.

*iii) Let* $E(S_i/\{R_i\}/R')$ *denote the relativistic structural entropy of* $S_i$ *given* $\{R_i\}/R'$.

*Then the relativistic structural entropy* $E(S/U\{R_i\}/R')$ *of the overall system* $S$ *given* $U_i\{R_i\}/R'$ *is given by the equation*

$$dE(S|U_i\{R_1\}/R')=\sum_{i1}^{n}dE(S_i/\{R_i\}/R'). \tag{28}$$

*For the proof, see Ref.* 6 *or* 16.

## PART II

## 9 AN APPROACH TO HIERARCHICAL DYNAMICS

### *9.1 Introductory Considerations*

At the present state-of-art, it has been recognized and it is definitively admitted that hierarchy appears to be one of the basic concepts of self-organization in general systems. This is quite understandable since a self-organizing system is one whose inner structure can be modified to get the best behavior with regard to its environment, and the hierarchical structure may be a way to convey this behavioral order inside the system. It follows that, in a certain sense, self-organization can be considered as equivalent to hierarchical dynamics, therefore the interest of an approach to the structural dynamics of hierarchies in general systems.

The main advantage of our relativistic structural entropy is to provide a description for hierarchical dynamics. Moreover by using the evolution principle, it yields a model for the study of these hierarchies.

### *9.2 A Quantum Model for Structural Dynamics*

#### *9.2.1 Problem and Solution*

i) *Notation* We shall use the notation $(S_1\rightarrow S_2)$ to emphasize that $(S_1/S_2)$ and $(S_2/S_1)$ compose a hierarchical system in which $(S_1/S_2)$ is the supremal element and $(S_2/S_1)$ is the infimal

one. The notation $(S_1, S_2)$ will be reserved for the case in which $(S_1/S_2)$ and $(S_2/S_2)$ are isolated from each other. $(S_1 \to S_2)$ means, for instance, $S_1$ controls $S_2$ or $S_2$ is the descendent of $S_1$.

ii) *Statement of the problem.* Consider three systems $(S_1/S_1)$, $(S_2/S_2)$ and $(S_3/S_3)$ which are first isolated from each other; then they compose the overall system $(S_1, S_2, S_3)$ whose structure is a decentralized one. When these systems are brought close to each other, they may spontaneously combine by themselves in a dynamical process composed of two stages. In the first stage they can yield one of the following combinations:

$$A: ((S_1 \to S_2), S_3); \quad B: ((S_2 \to S_1), S_3);$$
$$C: ((S_2 \to S_3), S_1); \quad D: ((S_1 \to S_2), S_3);$$
$$F: ((S_1 \to S_3), S_2); \quad G: ((S_3 \to S_1), S_2);$$
$$J: ((S_1 \to S_3), (S_2 \to S_3)); \quad L: ((S_3 \to S_1), (S_3 \to S_2));$$
$$M: ((S_2 \to S_1), (S_3 \to S_1)); \quad N: ((S_1 \to S_2), (S_1 \to S_3));$$
$$O: ((S_1 \to S_2), (S_3 \to S_2)); \quad P: ((S_2 \to S_1), (S_2 \to S_3)).$$

If the overall system reaches one of the structures $J$ to $P$ then it will keep this position. If the first structure is one from $A$ to $G$, then there is a second stage in which the two subsystems which compose the overall system may once more combine to provide one of the combinations $J$ to $P$, or one of the following hierarchical structures:

$$Q: (S_1 \to S_2 \to S_3); \quad T: (S_3 \to S_1 \to S_2);$$
$$U: (S_2 \to S_3 \to S_1) \quad V: (S_1 \to S_3 \to S_2);$$
$$W: (S_2 \to S_1 \to S_3); \quad X: (S_3 \to S_2 \to S_1).$$

Determine the structural trajectory that the overall system will so follow.

Remark on the physical meaning of the problem. For instance $D$ may be considered as the organization in which $S_1$ controls $S_2$, while $S_3$ remains isolated; in the structure $N$, $S_1$ controls both $S_2$ and $S_3$ while $S_2$ and $S_3$ are independent. Remark also that the topological feature of this problem comes from the fact that the model does not involve explicitly the time parameter; the structure of the whole system will change, but we need not to know when. The instance is similar to that of Markov chains.

iii) *General approach to the solution.* In the first stage, according to the evolution principle, among the configurations $A$, $B$, $C$, $D$, $F$, $G$, $J$, $L$, $M$, $N$, $O$, $P$, that one which will occur is the combination which has the greater direct relativistic structural entropy. We then have to compute these structural entropies and then select the greater one. Similarly, in the second stage, among the structure $Q$, $T$, $U$, $V$, $W$, $X$, the system will select that one which has the greater structural entropy.

Assumptions and notations. To get more insight in this structural dynamics, we make the following simplifying assumptions:

A1: The system $(S_i/S_i)$ has a constant objective $v(S_i/S_i)$, a constant transformation potential $w(S_i/S_i)$ and a constant organizability $\gamma(S_i/S_i)$ for every $i$.

A2: The organizability $\gamma(S_i/S_j)$ is also constant for every pair $(i,j)$ with $i \neq j$.

We point out that these assumptions are not merely working hypotheses, but are physically very realistic. If the components $S_1$, $S_2$ and $S_3$ are cell components, then they do not involve transformation potentials. Furthermore, a constant objective appears to be customary in nature; for instance, the objective of a living being would be to die (not to live). Assumption A2 may be regarded as a local assumption.

To simplify the writing, we put

$$H_0(i) \triangleq H_0(S_i/R'); \quad (i=1,2,3)$$
$$H_0(i/j) \triangleq H_0(S_i/S_j); \quad (i=1,2,3); \ (j=1,2,3); \ i \neq j$$
$$k(i) \triangleq K(S_i/R'); \quad (i=1,2,3)$$
$$\rho(i/j) \triangleq \rho(S_i/S_j); \quad (i=1,2,3); \ (j=1,2,3); \ i \neq j$$
$$\rho(i) \triangleq \rho(S_i/R'); \quad (i=1,2,3).$$

*9.2.2 Analysis of the first stage* To define the terminal structure of the first stage, we have to determine the direct structural entropies of the different possible combinations. As specific examples, we shall compute $E(A/A/R')$ and $E(J/J/R')$, the other ones are quite similar.

i) *Determination of* $E(A/A/R')$. Consider the subsystem $\Sigma \triangleq (S_1 \to S_2)$; $R'$ observes it by examining its outputs, in other words it observes $\Sigma$ via $S_2$; according the model $(S_1 \to S_2 \to R')$.

Then one has

$$dE(A/A/R')=K(S_2/R') \times\left[\frac{d\sigma(S_2/S_2)}{\sigma(S_2/S_2)}+\frac{d\sigma(S_1/S_2)}{\sigma(S_1/S_2)}\right],$$

and by using assumptions A1 and A2, (see Ref. [3])

$$dE(\Sigma/\Sigma/R')=K(S_2/R') \times\left[\frac{dH_0(S_2/S_2)}{H_0(S_2/S_2)}+\frac{dH_0(S_1/S_2)}{H_0(S_1/S_2)}\right],$$

therefore, by integrating

$$E(\Sigma/\Sigma/R')=\ln\left[H_0(2)^{k(2)}\,H_0(1/2)^{k(2)}\right].$$

In the same way, one has

$$E(S_3/S_3/R')=\ln H_0(3)^{k(3)}.$$

Since the systems $\Sigma$ and $S_3$ are in a parallel combination they are both directly observed by $R'$ and one has

$$E(A/A/R')=E(\Sigma/\Sigma/R')+E(S_3/S_3/R') =\ln\left[H_0^{k(2)}(2)H_0^{k(2)}(1/2)H_0^{k(3)}(3)\right]$$

ii) *Determination of* $E(J/J/R')$. By repeatedly using the two steps above, one has similarly

$$E(J/J/R')=\ln\left[H_0^{2k(3)}(3)H_0^{k(3)}(1/3)H_0^{k(3)}(2/3)\right].$$

iii) *Listing of the structural entropies.* We have:

$$E(A/A/R')=\ln\left[H_0^{k(2)}(2)H_0^{k(2)}(1/2 \times H_0^{k(3)}(3)\right] \tag{29a}$$

$$E(B/B/R')=\ln\left[H_0^{k(1)}(1)H_0^{k(1)}(2/1) \times H_0^{k(3)}(3)\right] \tag{29b}$$

$$E(C/C/R')=\ln\left[H_0^{k(3)}(3)H_0^{k(3)}(2/2) \times H_0^{k(1)}(1)\right] \tag{29c}$$

$$E(D/D/R')=\ln\left[H_0^{k(2)}(2)H_0^{k(2)}(1/2) \times H_0^{k(3)}(3)\right] \tag{29d}$$

$$E(F/F/R')=\ln\left[H_0^{k(3)}(3)H_0^{k(3)}(1/3) \times H_0^{k(2)}(2)\right] \tag{29e}$$

$$E(G/G/R')=\ln\left[H_0^{k(1)}(1)H_0^{k(1)}(3/1) \times H_0^{k(2)}(2)\right] \tag{29f}$$

$$E(J/J/R')=\ln\left[H_0^{2k(3)}(3)H_0^{k(3)}(1/3) \times H_0^{k(3)}(2/3)\right] \tag{29g}$$

$$E(L/L/R')=\ln\left[H_0^{k(1)}(1)H_0^{k(1)}(3/1) \times H_0^{k(2)}(3/2)H_0^{k(2)}(2)\right] \tag{29h}$$

$$E(M/M/R')=\ln\left[H_0^{2k(1)}(1)H_0^{k(1)}(2/1) \times H_0^{k(1)}(3/1)\right] \tag{29i}$$

$$E(N/N/R')=\ln\left[H_0^{k(2)}(2)H_0^{k(2)}(1/2) \times H_0^{k(3)}(1/3)H_0^{k(3)}(3)\right] \tag{29j}$$

$$E(O/O/R')=\ln\left[H_0^{2k(2)}(2)H_0^{k(2)}(1/2) \times H_0^{k(2)}(3/2)\right] \tag{29k}$$

$$E(P/P/R')=\ln\left[H_0^{k(1)}(1)H_0^{k(1)}(2/1) \times H_0^{k(3)}(2/3)H_0^{k(3)}(3)\right] \tag{29l}$$

By comparing the numerical values of these expressions for given initial states, we will obtain the configuration which will be achieved by $S_1, S_2$ and $S_3$.

iv) *A few comments.* Consider for instance the structures *A* and *O* pictured below

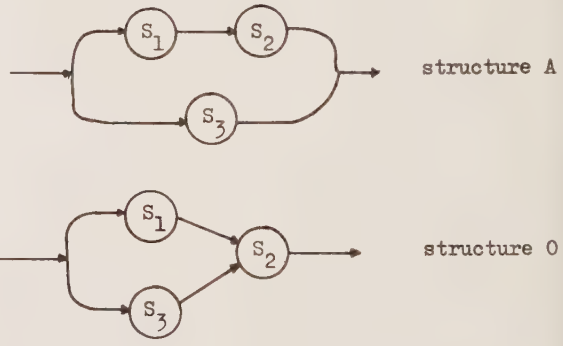



FIGURE 5 Structures *A* and *O*.

According to Eqs. (29a) and (29k) together with the evolution principle, if the following condition holds

$$H_0^{k(3)}(3)>H_0^{k(2)}(2)H_0^{k(2)}(3/2) \tag{30}$$

then the structure *A* will be realized. By contrast,

we will get the structure $O$ when

$$H_0^{k(3)}(3) < H_0^{k(2)}(2) H_0^{k(2)}(3/2). \tag{31}$$

Conditions (30) and (31) have a very interesting meaning. We have shown (see Ref. 2) that the structural entropy of a system, which is in the special case here $\ln H_0^{k(i)}(i)$, is the information potential of the system, that is to say the maximum amount of information disposable in the system for an external observer. It follows that, if the potential of $S_3$ is greater than that of $(S_3 \rightarrow S_2)$, then $S_3$ will remain isolated; but in the opposite case, $S_3$ will move to control $S_2$ and so, to get some further information from $S_2$. This result is quite understandable from a qualitative viewpoint, for instance, by considering $S_1$, $S_2$ and $S_3$ as three individuals in a given society; and so the question is to clearly define what we mean by the term *information* which must be identified for every kind of physical systems.

This being so, condition (31) is a necessary condition to get the model $(S_3 \rightarrow S_2)$ but it is not sufficient, since $S_2$ and $S_3$ can combine in the form $(S_2 \rightarrow S_3)$. By using calculations similar to those above, it is easy to show that this latter structure will occur only when

$$H_0^{k(2)}(2) < H_0^{k(3)}(3) H_0^{k(3)}(2/3),$$

and combining with (31) we obtain the necessary and sufficient condition for $(S_3 \rightarrow S_2)$, that is

$$H_0^{k(2)}(2)/H_0^{k(3)}(2/3) < H_0^{k(3)}(3) < H_0^{k(2)}(2) H_0^{k(2)}(3/2). \tag{32}$$

Similar considerations for the component $(S_1 \rightarrow S_2)$, in the structure $O$, yield the condition for $H_0^{k(1)}(1)$ that is

$$H_0^{k(2)}(2)/H_0^{k(1)}(2/1) < H_0^{k(1)}(1) < H_0^{k(2)}(2) H_0^{k(2)}(1/2). \tag{33}$$

We now have to write that $S_1$ cannot control $S_3$, and conversely, to avoid the structures pictured in Figure 6.

The respective direct structural entropies are,

$$E[(S_1, S_3)/(S_1, S_3)/R'] = \ln H_0^{k(1)}(1) H_0^{k(3)}(3)$$

$$E[(S_1 \rightarrow S_3)/(S_1 \rightarrow S_3)/R'] = \ln H_0^{k(3)}(3) H_0^{k(3)}(1/3)$$

$$E[(S_3 \rightarrow S_1)/(S_3 \rightarrow S_1)/R'] = \ln H_0^{k(1)}(1) H_0^{k(1)}(3/1).$$

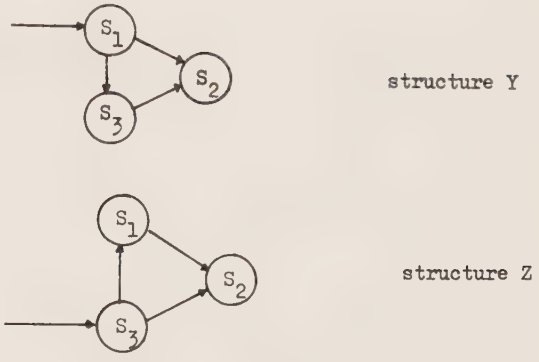



FIGURE 6 Two forbidden structures.

Comparing these entropies yields the following conditions:

$$H_0^{k(1)}(1) H_0^{k(3)}(3) > H_0^{k(3)}(3) H_0^{k(3)}(1/3) \tag{34a}$$

$$H_0^{k(3)}(3) H_0^{k(3)}(3) > H_0^{k(1)}(1) H_0^{k(1)}(3/1) \tag{34b}$$

They are equivalent to the following inequalities:

$$H_0^{k(1)}(1) > H_0^{k(3)}(1/3) \tag{35a}$$

$$H_0^{k(3)}(3) > H_0^{k(1)}(3/1) \tag{35b}$$

which can be combined in one, that is

$$\frac{\ln H_0(1/3)}{\ln H_0(1)} < \frac{k(1)}{k(3)} < \frac{\ln H_0(3)}{\ln H_0(3/1)}. \tag{36}$$

Condition (36) also has an interesting meaning. Indeed, assume that the organizabilities $u(S_1 R')$ and $u(S_3/R')$ have the same value, or equivalently

$$k(1) = k(3). \tag{37}$$

In such a case, Eq. (36) is always satisfied, as one has

$$H_0(1/3) < H_0(1)$$

and

$$H_0(3/1) < H_0(3).$$

In other words, if two systems $S_1$ and $S_2$, which are isolated from each other, have the same organizability $u(S_1/R') = u(S_2/R')$ then they will stay so in the future.

This statement depends explicitly upon the observer $R'$ in such a manner that two different

observers may conclude different behaviours for the system, and this is a direct consequence of the relativistic feature in our approach. So, the new question is the following:

> Given two systems $(S_1/S_1)$ and $S_3/S_3)$, is it possible to get initial conditions independent of the observer $R'$, which would determine the future structure of the overall system composed of $S_1$ and $S_3$?

This is possible when, and only when, the observer is one of the systems $S_1$ and $S_2$. Then, taking account of the equation

$$dH_i(S/S)=0 \text{ for every } S$$

(see Proposition 1), one has

$$E[(S_1\to S_3)/(S_1\to S_3)/S_3]=E(S_1/S_1/S_3)$$
$$=k(1/3)\ln \mathrm{H}_0(1),$$
$$E[(S_3\to S_1)(S_3\to S_1)/S_1]=k(3/1)\ln H_0(3),$$

therefore the following conclusion: assume that the two systems have the same external entropy; then the combinations $(S_1\to S_3)$ will occur whenever one has

$$k(1/3)>k(3/1), \tag{38}$$

the combination $(S_3\to S_1)$ will occur provided that

$$k(1/3)<k(1/1), \tag{39}$$

and the systems will remain isolated from each other, when

$$k(1/3)=k(3/1). \tag{40}$$

This result is quite understandable in terms of information potential. $E/S_1/S_1/S_3)$ is the information potential of $S_1$, considered as an information source, for $S_3$; in other words, the system on the whole will take the configuration which involves the larger amount of information.

### 9.2.3 *Analysis of the second stage*

*i) Determination of* $E(Q/Q/R')$. In this model $R'$ observes $S_3$ directly, it observes $S_2$ via $S_3$, and it observes $S_1$ via $(S_2/S_3)$. We then have

$$dE(Q/QR') = dE(S_3/S_3/R')+dE(S_2/S_3/R')$$
$$+dE(S_1/S_2/S_3/R'), \tag{41}$$

with

$$dE(S_1/S_2/S_3/R')=dI(S_1/S_2/S_3/R')/$$
$$\sigma(S_1/S_2/S_3/R'), \tag{42}$$

where $I(S_1/S_2/S_3/R')$ is the two-level relativistic information defined by Eqs. (18)–(20). It follows

$$dE(Q/Q/R')=K(S_3/R')\frac{d\sigma(S_3/S_3)}{\sigma(S_3/S_3)}$$
$$+K(S_3/R')\frac{d\sigma(S_2/S_3)}{\sigma(S_2/S_3)}+[K(S_3/R')\rho(S_2/S_1)$$
$$+K(S_2/S_1)\rho(S_3/R')]\frac{d\sigma(S_1/S_2}{\sigma(S_1/S_2)}. \tag{43}$$

Now, by using assumptions A1 and A2, we get

$$E(Q/Q/R')=\ln H_0^{k(3)}(3)H_0^{k(3)}(2/3)$$
$$\times H_0^{k(3)\rho(2/1)+k(2/1)\rho(3)}(1/2) \tag{44}$$

*ii) Listing of the structural entropies.*

$$E(Q/Q/R')=\ln [H_0^{k(3)}(3)H_0^{k(3)}(2/3)$$
$$\times H_0^{k(3)\rho(2/1)+k(2/1)\rho(3)}(1/2)] \tag{45a}$$

$$E(T/TR')=\ln [H_0^{k(2)}(2)H_0^{k(2)}(1/2)$$
$$\times H_0^{k(2)\rho(1/3)+k(1/3)\rho(2)}(3/1)] \tag{45b}$$

$$E(U/U/R')=\ln [H_0^{k(1)}(1)H_0^{k(1)}(3/1)$$
$$\times H_0^{k(1)\rho(3/2)+k(3/2)\rho(1)}(2/3)] \tag{45c}$$

$$E(V/V/R')=\ln [H_0^{k(2)}(2)H_0^{k(2)}(3/2)$$
$$\times H_0^{k(2)\rho(2/1)+k(2/1)\rho(2)}(1/2)] \tag{45d}$$

$$E(W/W/R')=\ln [H_0^{k(3)}(3)H_0^{k(3)}(1/3)$$
$$\times H_0^{k(3)\rho(1/2)+k(1/2)\rho(3)}(2/1)] \tag{45e}$$

$$E(X/X/R')=\ln [H_0^{k(1)}(1)H_0^{k(1)}(2/1)$$
$$\times H_0^{k(1)\rho(2/3)+k(2/3)\rho(1)}(3/2)] \tag{45f}$$

The determination of the structure which will be achieved may be done as in the first stage, by

comparing the respective structural entropies. For instance, by using Eqs. (29f) and (45d), a necessary (but not sufficient) condition in order that the system move from $G$ to $U$ is

$$H_0^{k(1)\rho(3/2)+k(3/2)\rho(1)}(2/3) > H_0^{k(2)}(2). \qquad (46)$$

If the organizabilities are low, and if

$$H_0^{(2/3)} = H_0(2),$$

then condition (46) is approximately equivalent to

$$u(S_1/R') + u(S_3/S_2) > u(S_2/R')$$

which is understandable from a practical viewpoint.

## 10 DYNAMICS OF FORMS

### *10.1 General Approach*

The model in Section 9 can serve to address the problem of morphogenesis considered as a growing of forms. Two main ideas are the bases of this approach: (i) a form may be pictured as a hierarchical system and (ii) the dynamics of a form is basically a local dynamics. What follows is a brief outline to this question.

Consider four systems ($S_1$, $S_2$, $S_3$, $S_4$) denoted by (1, 2, 3, 4) which may combine by themselves to yield an overall structure. This combination is carried out in three steps: in the first one, two systems are put together (Figure 8); and in the last one, the terminal structure is achieved by the four systems. If these systems are physically similar, the different hierarchies that they so yield are equivalent to the various forms which are pictured in Figures 7 to 10. We then have a

FIGURE 7 Systems 1, 2, 3, 4 isolated each other.

kind of isomorphism between hierarchies and forms, and the determination of the form trajectory which will be followed is equivalent to determine what hierarchy structure will be selected by the system. This problem has been solved in the preceding section.

FIGURE 8 First stage of the combination of 1, 2, 3, and 4.

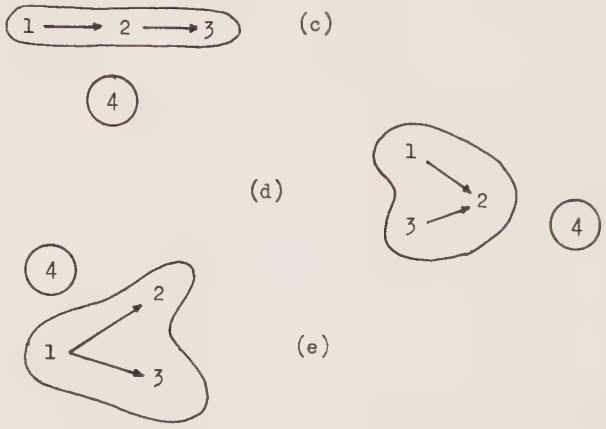



FIGURE 9 Second stage of the combination of 1, 2, 3, and 4.

### *10.2 Mathematical Considerations*

In view of Figures 7 to 10, it is not evident why the patterns (c) and (d) should be different, since the latter may be pictured in the form (1→2←3) which is linear like (c); and we so have to define exactly what we mean by the expression "similar forms". We are so led to the concept of homeomorphism as introduced by mathematicians. Qualitatively speaking, two forms $F_1$ and $F_2$ are similar, say homeomorphic provided that one can continuously modify $F_1$(or $F_2$) to obtain $F_2$ (or $F_1$). With this definition, there would be only two different forms in the models (c) to (1) and they are pictured in Figure 11. The model (n) is quite similar to the Riemann-Hugoniot curve which is so pervasive in physical science.

We believe that these preliminary considerations could be expanded to obtain a connection with the catastrophe theory of Thom.[1] Indeed, the graphical representation in space of certain relations holding in a sytem can be considered as

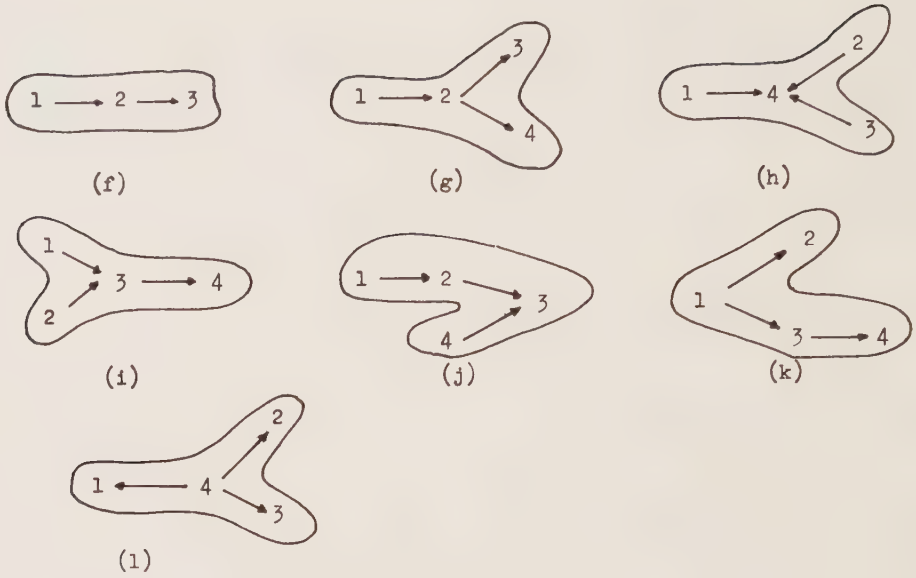



FIGURE 10 Third stage of the combination of 1, 2, 3, and 4.

FIGURE 11 Models equivalent to those (*c*) to (l).

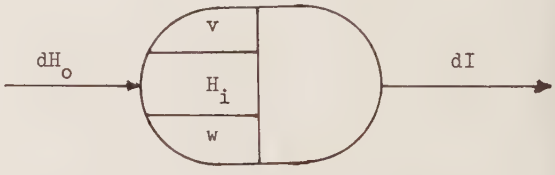



FIGURE 12 General model for general systems.

equivalent to the arrangement of the elements which compose a system, and we think more especially to the arrangement of the molecules inside a crystal. In fact our assumption would be to consider a form as the physical representation of a hierarchical system, and perhaps in this way we would obtain the physical support of the catastrophe theory.

## 11 A FEW EXAMPLES

Our purpose, in this section, is to illustrate how the general model above can be used to describe concrete systems. In fact, the main difficulty is to identify the physical meaning of the abstract variables above, and to this end a few general considerations may be helpful.

The model is pictured in Figure 12. The system received an amount of knowledge or, similarly, its own information $dH_0$ from its environment. It uses this information to modify its internal variables $v$, $H_i$ and $w$; and then it pours the amount $dI$ of information into the environment. Remark on the difference between the informations $H_0$ and $I$: $H_0$ is an information from the viewpoint of the system, while $I$ is an information from the viewpoint of the environment. In other words, any variable which is the input of the system can be considered as being $H_0$ or else any variable for which the system presents a certain sensitivity. In this way, if we have two systems $S_1$ and $S_2$ in the cascaded combination $(S_1 \rightarrow S_2)$, then the output $dI_1$ of $S_1$ is the input of $S_2$ (Figure 13), that is:

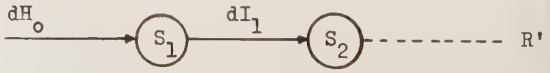



FIGURE 13 Input–output in cascaded systems.

$$d\sigma^2(S_1/S_2)=c^2dH_0(S_1/S_2)-dH_i^2(S_1/S_2)$$
$$-dv^2(S_1/S_2)-dw^2(S_1/S_2)$$
$$dI(S_1/S_1/S_2)=(\rho u/c)_{S_1/S_2}$$
$$\times[d\sigma(S_1/S_2)/\sigma(S_1/S_2)]$$
$$d\sigma^2(S_2/R')=c^2dI^2(S_1/S_1/S_2)-dH_i^2(S_2/R')$$
$$-dv^2(S_2/R')-dw^2(S^2/R')$$

This being so, we can outline some examples.

*11.1 Energizing Physical Systems*

We consider a pure physical system, from the viewpoint of energy transfer. One may assume that $dv=dw=0$, or else, with a suitable origin, $v=w=0$, so that

$$d\sigma^2=c^2dH_0^2-dH_i^2.$$

The system is subject to the total amount $c^2dH_0^2$ of energy; a part $dH_i^2$ of this energy is used to modify the internal structure of the system, so that the amount of energy which is disposable for an external user is the difference $d\sigma^2$. This phenomenon is well known in electricity, for instance, where $dH_i^2$ is named the reactive energy, and $d\sigma^2$ the active energy. If we write $d\sigma^2$ in the form

$$d\sigma^2=c^2dH_0^2\left(1-\frac{1}{c^2}\frac{dH_i^2}{dH_0^2}\right),$$

we get exactly the term $\cos\phi$, which is customarily used by electricians who write

$$P=UI\cos\phi$$

where $P$ is the electric power. The entropy of the system is

$$E(S/S/R')=K(S/R')\ln\sigma(S/S)$$

and the energy available in the system will increase, provided that $K(S/R')$ is constant with

$$K(S/R')>0. \qquad (47)$$

But $K(S/R')$ is exactly the efficiency of the information transfer from $S$ to $R'$; (47) is then always satisfied and $\sigma(S/S)$ increased. Remark that $\sigma(S/S)$ is the total amount of energy that the system stores.

Remark also the analogy with the thermodynamical entropy $S$ defined by its differential

$$dS=dQ/T$$

where $dQ$ represents an infinitesimal amount of energy transfer as heat, and $T$ is the absolute temperature of the medium. For a constant volume one has

$$dS=c\,dT/T$$

where $c$ denotes the specific heat of the medium; and we then have the straightforward identification

$$dQ=K(S/R)d\sigma(S/S)$$
$$cdT=K(S/R)dH_0(S/S).$$

*11.2 Differential Equations*

Consider the scalar differential equation

$$\dot{x}(t)=f[x(t),t],\qquad x(0)=x_0.$$

If we define the input of the system as being time $t$, we then have the straightforward identification

$$H_0=t \qquad H_i=t$$
$$dv=0 \qquad dw=0$$

therefore the metric

$$d\sigma^2=c^2\left(1-\frac{f^2}{c^2}\right)dt^2$$

where $c$ denotes the light velocity. Since $f$ is small *w.r.t.* $c$ in this Galilean model, one has

$$f/c\cong 0$$

therefore

$$dE\cong\frac{f}{c}\frac{dt}{t}=0$$

and the system evolves with a constant structural entropy.

*11.3 Optimum Control Systems*

Let now be given the optimum control problem

$$\dot{x}(t)=f(x,a,t), \qquad x(0)=x_0$$

$$\max_a G(a)=\int_0^T g(x,a,\sigma)d\sigma$$

where $T$ has a fixed value. The disturbing question here is how to consider the maximization w.r.t. $a$: either we define it as the objective of the system or else as the maximization of the structural entropy of the system. Since the system always maximizes the same functional, one can assume that $v$ is constant, and identifying the maximization of $E$ with that of $G$ provides

$$dE=\mathscr{H}\,dt$$

where $\mathscr{H}$ denotes the Hamiltonian of the system. If we further assume that

$$dE(S/S/R')=dE(S/S)$$

we then get the metric

$$d\sigma^2=e^{2\mathscr{H}}d\mathscr{H}^2. \tag{48}$$

The reader might consider these comments purely speculative, but in fact they may have some interest in multilevel hierarchical control systems where the input of an infimal level is the $d\sigma$ of the supremal level just above. Moreover, the identification of the state $\sigma$ with the Hamiltonian is consistent with other results that we have obtained before in optimum filtering;[17] the best filter is that one which gives the best estimate of the Hamiltonian rather than that of the position $x$.

*11.4 Stochastic Processes*

The concepts of internal entropy and external entropy hold also for stochastic processes. Consider two independent random variables $X$ and $Y$ with the probability density functions $p(x)$ and $q(y)$ and let $Z=X+Y$ denote their sum.

Usually the entropy $H(Z)$ of $Z$ is defined as follows: one first determines the density function $r(z)$ of $z$, that is

$$r(z)=\int_{-\infty}^{+\infty} p(z-y)q(y)dy, \tag{49}$$

and then one uses the equation

$$H(Z)=-\int_{-\infty}^{+\infty} r(z)\ln r(z)dz.$$

Now, from the relativistic viewpoint, an observer $R'$ may consider $Z$ either for its external value $Z$ or for its internal structure $(X,Y)$. In the first case, the uncertainty is related to $Z$ itself, but in the second the uncertainty concerns the values of $X$ and $Y$ which compose $Z$. In this way, the external entropy of $Z$ is $H(Z)$ as defined by Eq. (50) while its internal entropy is the Shannon entropy $H(X,Y)$ of the pair. Assume now that the above probability functions depend on time, the metric of the process would be

$$d\sigma^2(X,Y,t)=c^2dH^2(Z,t)-dH^2(X,Y,t)$$

and the system would evolve in order to maximize $\sigma(t)$. This approach would be a way to attack the following problem, which may be considered as a preliminary model for a broad class of questions as for instance in biology.

Consider $n$ independent random variables $X_1,X_2,\ldots,X_n$ with their probability density functions $p_i(x)$. Among these, two variables say $X_i$ and $X_j$, can spontaneously combine to yield the sum $X_i+X_j$. Determine the pair $(X_i,X_j)$. Remark that this statement does not involve explicitly the time. Indeed, it is not certain at all that time is the central variable in biological systems as in dynamical systems: and more and more,[1,5] one is inclined to think that it plays an auxiliary role in these systems, in the sense that it is not an input, but simply a practical subscript to describe the evolution. □

Our working assumption is that the system would select the pair $(X_i,X_j)$ which has the higher structural entropy $E(X_i,X_j)$. Since one has

$$dE(X_i,X_j)=d\sigma(X_i,X_j)/\sigma(X_i,X_j),$$

the system would evolve in order to maximize $\sigma(X_i,X_j)$. □

Remark that the problem is meaningless for a set of two independent variables only; but, in such a case, one can investigate, for instance, the dynamics of the probability functions when they depend on time. The result so obtained is not new, but is consistent with existing results.

Consider, for instance, two random variables $X_1$ and $X_2$, where $X_1$ takes on values 1, 2, 3 with the probabilities $p_i(t)$, $i=1,2,3$, *and* $X_2$

takes on values $-1$, $-2$, $-3$ with the probabilities $q_i(t)$, $i=1, 2, 3$; furthermore $p(X_1=k/X_2=-k)=1$ for $k=1, 2, 3$ and all $t$.

$$H(Z,t)=0 \text{ and } H(X_1, X_2, t)=H(X_1, t),$$

therefore

$$d\sigma^2(t)=-dH^2(X_1, t)$$

and the structural entropy is given by the equation

$$dE(t)=dH(X_1, t)/H(X_1, t$$

therefore

$$E(t)=\ln H(X_1, t).$$

According to the evolution principle, the system will evolve in order to maximize its structural entropy. In other words, $H(X_1, t)$ will tend to its maximum value, i.e. the distribution of $X_1$ will tend to the uniform one $p_1=p_2=p_3=1/3$. This result is well known in statistical thermodynamics.

### *11.5 Linguistics*

Broadly speaking, a linguistic system is composed of a language and a universe of discourse. The language $L$ itself starts from symbols and phonemes which are arranged in words, sentences, subsections, sections . . . in accordance with certain rules which are customarily called grammar rules or syntax. The universe of discourse $W$, which may be a physical one or a set of philosophical concepts, is in correspondence with the language via a certain isomorphism, named the semasiology or the semantics of the language.[18]

We then straightforwardly have the following identification (cf. Figure 14). ($H_i \equiv$ syntax) and ($H_0 \equiv$ semantics). Assume now that $H_i$ and $H_0$ are varying, as in a learning process for instance, then the language will evolve in order to maximize $\sigma$, provided that the learning coefficient $K(L/R')$ is constant. The question now is to attribute numerical values to $H_i$ and $H_0$. This could be done by remarking that the basic role of language is to transfer information, so that $H_i$ and $H_0$ may again have a meaning of uncertainty.

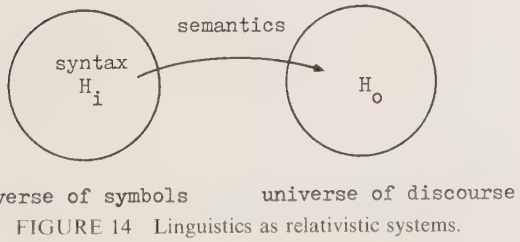
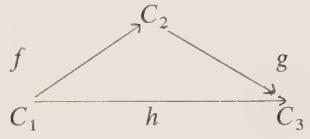


FIGURE 14 Linguistics as relativistic systems.

Such an approach could be of interest in investigating the evolution of languages, the learning process of languages and automatic translation systems.

### *11.6 Societal Systems*

The basic feature of societal systems is the existence of relationships between the individuals or the classes of the considered society. These relationships can be unified and simplified by a representation with diagrams of arrows. Each arrow $f: C_1 \to C_2$ represents a function, that is to say two classes $C_1$ and $C_2$ and a rule $S_1 \mapsto f(S_1) = S_2$ which assigns to each system $S_1 \in C_1$ a system $S_2 \in C_2$. A typical diagram of classes and functions is




which is commutative when $H(S_1)=g \circ f(S_1)$.

The relation $C_1 \to C_2$ may be considered, for instance: "$C_2$ is hierarchically under $C_1$", "every man belonging to $C_1$ is the husband of a wife in $C_2$", "$S_2$ is a relative of $S_1$", etc.... In our relativistic framework this diagram has the representation

$$C_1 \xrightarrow{f} C_2 \xrightarrow{g} C_3 \equiv (C_1/C_2/C_3)$$
$$C_1 \xrightarrow{h} C_3 \quad \equiv (C_1/C_3)$$

where the relational functions $f$, $g$ and $h$ appear in the different relativistic coefficients $K(C_i/C_j)$. For instance, one has

$$dE(C_1/C_3)=K(C_1/C_3)\frac{d\sigma(C_1/C_1)}{\sigma(C_1/C_1)}$$

therefore one can make the identification

$$h=K(C_1\ C_3)$$

and in the same way, multilevel relativistic information provides the identification

$$g \circ f = K(C_1/C_2)\rho(C_2/C_3) + \rho(C_1/C_2)K(C_2/C_3). \quad (52)$$

So, if one considers that the above diagram is the point of departure of the category theory[19] which can be a mathematical tool to describe the structure of societal systems, then our relativistic formulation could be a way to investigate the structural dynamics of categories.

### *11.7 Biological Systems*

One of the most fruitful ideas in biology is the fascinating concept of the *gene* which can be roughly defined by its following properties. The gene is hereditary, it contains coded information, it occupies specific discrete places located on chromosomes, it can mutate to give alleles, combinations of chromosomes can occur via the gene loci.

In our relativistic framework, the basic question is the following: what exactly do we mean by the terms of coded information in the gene $G$? It could be the information potential or, similarly, the direct structural entropy $E(G/G/R')$ but what is the meaning of $R'$? In the chain

$$(\text{DNA} \rightarrow \text{RNA} \rightarrow \text{Amino Acid}) \equiv (D \rightarrow R \rightarrow A),$$

which conveys the information in question, we should consider somewhat relativistic information as

$$dI(D/R/A) = K(R/A)d\sigma(D/R)$$

in which $\sigma(D/R)$ denotes the initial information potential and $K(R/A)$ is the capacity of the channel which transfers this information. If $K(R/A) < 1$, then there is a loss of information in the transfer. It is well known that this is effectively the case since we have only 20 amino acids while 64 would be theoretically available; but then, what is truly the physical meaning of $K(R/A)$? Answering this question would be very interesting as the use of multilevel relativistic information, and its composition law for $K$ would permit suggestions for further research.

## 12 CONCLUSIONS

The present theory, outlined in this paper, is one of numerous attempts to identify a general theory for general systems. Based on the remark that any information $I$ involves the system $S$ which pours this information and the observer $R'$ who uses it, therefore the necessity to consider the triplet $(S, I, R')$, we have simultaneously obtained a relativistic description for general systems and a new relativistic information theory involving both syntax and semantics. There remains much to do, but at the present state-of-art the main results related to the theory may be summarized as follows. We have shown[6] that relativistic information yields Shannon information under assumptions which are transparent in this theory; it provides fuzzy set theory as a special case (therefore a new look at this latter); and it yields a new relativistic interpretation for probability, which will solve, perhaps, some paradoxes in quantum mechanics as that of the "Schrodinger's cat" for instance.

From the viewpoint of general systems, there are two basic contributions: first, the use of internal entropy and external entropy provides a description for the well known closing-opening duality in general systems, and second, the relativistic formulation is quite well adapted to describe dynamical hierarchies in organization.

In fact, it is quite likely that two systems which are different from a physical point of view, say a biological system and a societal one, have two different metrics as

$$d\sigma_1^2 = g_1 dH_0^2 - g_2 dH_i^2 - g_3 dv^2 - g_4 dw^2$$

$$d\sigma_2^2 = h_1 dH_0^2 - h_2 dH_i^2 - h_3 dv^2 - h_4 dw^2$$

where the $g_i$'s and the $h_i$'s denote given functions depending on the natures of the two systems; for instance, they may represent various physical constraints on the system. In such a case, the Lorentz transformation would not hold, and all we can expect is that the two metrics $d\sigma_1^2$ and $d\sigma_2^2$ are locally equivalent. We then have the general theory allowing us to investigate the interaction of different systems.

For the sake of brevity we deleted some interesting questions concerning the observer himself. Indeed, an observer $R$ is a system, and consequently has an internal structure which may be modified, so that the question is how to

introduce this dynamics in the formalism. In fact, this internal structure is explicitly contained in the coefficient $K(S/R)$ via the external entropy $H_0(S/R)$ and one may expect that an approach to this problem could be provided by applying the evolution principle to the overall system composed of $S$ and $R$. If we consider the structural entropy as an information potential,[6,21] it would remain to investigate the dynamics of $\sigma(R/S)$ subject to this potential.

Another interesting question is the interference of two observers $R$ and $R'$ with one another, given a same system $S$. According to Section 4, the external $H_0(S/R, R')$ is generally different from $H_0(S/R)$ and $H_0(S/R')$ so that the observed metric itself is affected by this interference, and there remains the task of estimating this effect.

We intend to investigate these questions later.[22,23,24,25]

REFERENCES


1. R. Thom, *Stabilité Structurelle et Morphogenèse* (Structural stability and morphogenesis). W. A. Benjamin, Reading, Massachusetts, 1975.
2. M. Davis, *Computability and Unsolvability*. McGraw-Hill, New York, 1958
3. O. Morgenstein, On the theory of economics, the theory of accurateness and the normative theory. *Kyklos Review*, Kyklos-Verlag, **25,** No. 4, 1972.
4. G. Jumarie, A relativistic information theory model for general systems. Lorentz transformation of organizibility and structural entropy. *International Journal of Systems Sciences* **6,** No. 9, 1975, pp. 865–886.
5. G. Jumarie, Further advances on the general thermodynamics of open systems via information theory: effective entropy, negative information. *International Journal of Systems Sciences*, **6,** No. 3, 1975, pp. 249–268.
6. G. Jumarie, *Théorie Relativiste de l'Information. Application à un Modèle Général d'Evolution des Systèmes* (Relativistic Information Theory. Application to a General Model for the Dynamics of General Systems). Special issue of the *Revue du CETHEDEC* (Journal of the Center for Theoretical Studies of Detection and Communication), **13,** No. NS76–1, 98 pages, Paris, 1976.
7. S. A. Eddington, *The Mathematical Theory of Relativity*. Cambridge University Press, Cambridge, 1963.
8. E. Pinel, *Les Fondements de la Biologie Mathématiques non Statistique* (Foundations of non statistical mathematical biology). Maloine, Paris, 1973.
9. E. Pinel, *La Relativité en Biology* (Relativity in Biology). Maloine, Paris, 1975.
10. C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, Ill., 1949.
11. S. Guiasu and R. Theodorescu, *La Théorie Mathématique de l'Information* (The mathematical theory of information) Dunod, Paris, 1968.
12. J. Wolfowitz, Information theory for mathematicians. *Annals of Mathematical Statistics*, **29,** pp. 351–356.
13. J. Aczel and Z. Daroczy, *On Measures of Information and their Characterizations*. Academic Press, New York, 1975.
14. S. Guiasu, *Information Theory with New Applications*. McGraw-Hill, New York, 1977.
15. *Proceeding of the VIIIth International Congress on Cybernetics*, Symposium "on the different aspects of the concept of information", Namur, 1976, also in *Cybernetica*, **XX,** No. 1, pp. 11–20, by Jumarie, 1977.
16. G. Jumarie, A relativistic information approach to the structural dynamics of general systems. *Cybernetica*, **19,** No. 4, 1976, pp. 273–304.
17. G. Jumarie, A new approach to optimum state estimation of optimum system control. *Journal of Optimization Theory and Applications*, **16,** No. 1–2, 1975, pp. 135–146.
18. G. Jumarie, Some technical applications of relativistic information. Probability, fuzzy sets, linguistics and transmission problems. *Cybernetica*, **20,** No. 2, 1977, pp. 91–128.
19. S. Mac Lan, *Categories for the Working Mathematicians*. Springer Verlag, New York, 1971.
20. G. Jumarie, Structural entropy, information potential, information balance and evolution in self-organizing systems. *International Journal of Systems Sciences*, **5,** No. 10, 1974, pp. 953–972.
21. G. Jumarie, Further advances in relativistic information and general systems. Interference of observers. *Cybernetica*, 21, No. 2, 1978, pp. 93–123.
22. G. Jumarie, Théorie relativiste de l'information et télécommunication. Perspectives. (Relativistic information theory and communication. Applications.) *Annales des Télécommunications*, **33,** No. 1–2, 1978, pp. 13–27.
23. G. Jumarie, A survey of relativistic information and its applications. In: *Information and Systems*, edited by Dubuisson, Pergamon Press, New York, 1978, pp. 121–126.
24. G. Jumarie, Relativistic fuzzy sets as a mean to introduce human factors in pattern recognition systems. *Proceeding of IEEE International Conference on Cybernetics and Society*, Tokyo, 1978.
25. G. Jumarie, Théorie relativiste de l'information II. Information de Shannon, entropie de Rényi, ensemble flou relativiste. (Relativistic information theory II. Shannon information, Renyi entropy, relativistic fuzzy set.) *Annales des Télécommunications*, to appear, 1979.
26. G. Jumarie, *Subjectivité, information, système: Synthèse pour une cybernétique relativiste* (Subjectivity, information, system: A unified appraisal to a relativistic cybernetics). Univers-Aurore Editeurs, Montréal, 1979.

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# *Book Reviews*

*The editor invites books for review on any subject relevant to general systems. Books for review must be sent to Dr. George Klir, Editor,* International Journal of General Systems, *Department of Systems Science, School of Advanced Technology, State University of New York, Binghampton, New York 13901, U.S.A.* (*and* not *to the publisher*).

## SYSTEMS ENGINEERING METHODOLOGY FOR INTERDISCIPLINARY TEAMS, by A. Wayne Wymore, John Wiley and Sons, New York, 1976. 431 pp.

Professor Wymore's book came to fill a notable vacuum in the growing body of systems engineering literature. To this reviewer's knowledge, the approach proposed in this book represents the first truly systemic approach to large-scale complex, man/machine system design. It rests on a solid foundation of mathematical system theory and its principal aim is to guarantee that the end product of the design effort will be indeed a system. At the same time, the proposed approach stays as close as practicable to most intuitive and several methodological approaches and utilizes many concepts from several specialized methods. It is important to state this at the outset because there is a danger for the unfamiliar reader to get lost in some of the formalisms peculiar to this methodology and lose sight of its flexibility and generality.

The author states that this book "is intended to be read by any (and all) professionals who aspire to participate effectively in the work of interdisciplinary teams devoted to the design or analysis of large-scale, complex, man/machine systems". This statement sets the tone for a very ambitious undertaking which, to be successful, requires that readers be willing to match Professor Wymore's enthusiasm step by step. And although it is strictly true that no specific level of technical sophistication is required for reading it, those with a quantitative technical education will have a definite advantage becoming proficient at using this methodology to tackle real problems. This observation is important concerning the use of this book as a textbook. Class composition can be a significant factor. So is the time available to absorb the methodology and acquire sufficient proficiency to apply to realistic examples (I recommend a six-credit hours course at the beginning graduate level). It is also important to realize that Professor Wymore does not interpret the word methodology to mean a collection of handy tools and techniques but rather as a set of conceptual guidelines implementable by a variety of techniques. Hence, less experienced users of this methodology will not find it easy to put into practice without some additional guidance.

Chapter 1 is an excellent exposition of systems engineering as a distinct engineering discipline: the problems it is concerned with, the people it should bring together to solve them, the foundations for the different approaches and methods used by these people. The concept of an interdisciplinary team and its role in the large-scale, complex, man/machine problem solving enterprise is introduced as a pivotal element. Then, the need for a system design methodology which does not start from an assumed solution form is argued. This, in fact, is perhaps the central issue which provides motivation for Professor Wymore's rigorous methodological approach. This does not mean that the proposed methodology is not solution oriented. Quite the contrary; the potential success of the approach rests on constructing a very comprehensive problem definition statement which is normative in nature. That is, to define the problem under study we are asked to provide a complete set of specifications for solutions (systems) to such problems. But, we are also asked to avoid committing ourselves prematurely to any one specific solution or class of solution or class of solutions. Another important issue is the need to adopt a common, precisely defined language to which every member of this interdisciplinary team subscribes for the sake of maintaining effective communications throughout the system design process.

Heuristic definitions of the basic artifacts of

this systems design methodology are given in Chapter 1. Then (Section 1.5) the concepts of system, and system models are reviewed. This chapter ends with an extensive section on manipulation of systems where a few surprises will greet readers familiar with other system design and systems analysis literature. For example, definitions of subsystems and system simulation are not conventional and this fact better be kept in mind while studying subsequent chapters, because both these concepts play key roles in applications of this methodology.

Chapter 2 begins with a thorough yet compact review of literature relevant to system design methodology. This is a most useful review because it helps to make connections between Wymore's mathematically based approach and more intuitive approaches. This reviewer recommends that readers go back to this literature survey after they have become familiar with Wymore's methodology as a means to gain further insight into the compatibility between his and other approaches.

The bulk of Chapter 2 is concerned with input/output specifications, the "*what* needs to be done" portion of a problem definition statement. Systems analysts will recognize the equivalence between the popular "black box" concept and the input/output specification concept developed by Professor Wymore. On the other hand, the unfamiliar reader could use a bit more guidance about what inputs (outputs) are meant to be included in an input/output specification and what inputs (outputs) are not. In particular, there is no adequate discussion about what I would call "environmental" inputs and outputs (physical, legal, administrative, etc.), as distinct from "basic purpose" inputs and outputs (people, things to be processed by systems).

There is considerable room for refinement and imaginative choice of more practical displays vis-a-vis the type of input/output specification formats adopted by Professor Wymore. Even for simple problems, these formats can get quite cumbersome. In this book, he has emphasized the basic contents of this and the other five principal artifacts of the problem definition statement, without being too concerned about the practicality of the formating tools he has chosen to display the relevant information. The challenge for users of this methodology is then to come up with more practical formats, particularly when dealing with problems of realistic levels of complexity.

Another significant issue brought to focus by a review of Chapter 2 is the appropriate breadth and depth of the material that should be covered the first time through. In the present case, with considerable classroom testing hindsight, this reviewer is of the opinion that a better tactic would have been to have a briefer run through the entire methodology than that presented in Chapters 2 to 5 and then have additional chapters dealing with special cases and more advanced material.

At this point it seems appropriate to digress about implementability of a rigorous methodology such as this. It is clear that Prof. Wymore is aiming ultimately at a highly mechanized (i.e. computerized) system design process such that large numbers of valid solutions may be generated and evaluated comparatively to select an optimal solution to the problem at hand. To make such mechanization possible it is desirable that all items of information to be considered in the solution-seeking process be standardized and as free of ambiguity as possible. On the other hand, for less mechanized processing there is considerable freedom concerning formats and displays within the constraints imposed by the type of information that should be handled and the level of detail desired. The point is that one should not lose sight of the essence of the methodological rigor while attempting to cope with the complexities of particular information recording and processing formats. If this point is taken as a warning throughout this book, one has a far better chance to perceive and be in a position to utilize the conceptual power of Wymore's approach.

Chapter 3 contains an excellent discussion on the "*how* we can do it" portion of the problem definition statement. This includes a definition of "a technology" and a valiant attempt to define implentability of a system in non-mathematical language. I doubt whether anybody can do it better than Prof. Wymore, but judging from the material presented in his book, checks on system implentability might have to remain in the domain of tasks to be performed by the trained system engineering members of the interdisciplinary team. I am perfectly comfortable with this division of labor, and I suspect systems engineers will want to develop increasingly efficient methods for carrying out this intrinsically cumbersome verification job.

Chapter 4 is concerned with merit orderings,

i.e., the methodological artifacts necessary to compare different problem solutions to one another. The reader will find here some familiar concepts (e.g. probability distributions, figures of merit, performance indices) given precise, mutually consistent definitions. The concept of a "system experiment", defined in Chapter 1 is the basis for assessing system behavior so as to enable comparison of any two systems. Of particular interest here is the precise definition of trade-off between input/output-related performance and technology-related performance.

Professor Wymore emphasizes that defining merit orderings appropriately is the most important task of the interdisciplinary design team. The central idea is not to attempt to model the client's value system but rather to define and document the negotiated basis for comparing any two solutions to the problem at hand. Hence, definitions of merit ordering artifacts are solution oriented, a presccription well illustrated in this chapter.

Several important points are brought out in Chapter 5. Perhaps the key point is that system testing should be considered an essential part of the problem definition statement negotiated with the client. Different questions concerning the relation between a system model and a real-life implentation of such a model are discussed therein. Finally, this chapter contains a discussion of the relation between system testing and the merit orderings defined in Chapter 4.

Having completed the definition of the methodological artifacts of a system design problem statement, Professor Wymore devotes the last chapter in the book to a discussion of the process of system design. Concerning proposed solutions to the stated problem, a brief discussion about justification of system components is offered. Then synthesis is viewed as the process of developing system theoretic models with increasing levels of detail. The last important discussion in this chapter deals with system function analysis and contains many suggestions for future work directions. This chapter is rather brief. The level of methodological detail is much lower than that at which problem definition is treated earlier in the book. There are no illustrative examples with any content. Hopefully, the challenge of developing adequate methodological artifacts for carrying out the detailed work of generating and evaluating solutions will be answered by capable systems engineers in a near future. This book should provide a solid point of departure for such work.

The book concludes with an Appendix where the mathematics of tricotyledon theory of system design is presented by way of a set of definitions. This report on the theoretical foundations of the methodology is far from being sufficient for self tutorial purposes. However, it does provide a complete mathematical sketch of the theory, which should help clarify many of the fine points of the methodology. Besides, a more detailed exposition of tricotyledon theory is not yet available in the open literature and hence, the terse treatment it gets here will have to do for the present.

In view of some recent trends towards more rigor and more completeness in system design practice, this book must be considered a very significant and welcome addition to current systems engineering literature. One can find many points of contention concerning choice of content, formats, scope, and distribution of material. However, the basic postulates on which the proposed methodology rests seem to be very sound and should serve as a strong foundation for future developments in both theoretical and applied areas of system design methodology. If readers will resist a premature temptation to "fight" Professor Wymore's unique approach and style of presentation they are in for a very rewarding learning and mind opening experience.

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EVOLUTION AND CONSCIOUSNESS: HUMAN SYSTEMS IN TRANSITION, Erich Jantsch and Conrad H. Waddington (Editors), Addison-Wesley, Reading, Mass. 1976. 260 pp.

In this volume thirteen authors—under the editorship of Erich Jantsch, from Berkeley, and the late Conrad Waddington, from Edinburgh—update Genesis and Darwin into an operational "conscious design of life within a fluidity of transformation". The essays exude the spirit of our time and particularly those written by Prigogine and Pankow illuminate it with Promethean fire. Jantsch and Waddington have assembled a colorful coterie of original

thinkers who all expect, praise and welcome dissipative non-equilibrium, self-organizing structures, in short, the Heraklitean change implicit in our perception-conception of life, death and renewal.

Clearly the state of art in science, our technology and social structure have supplied the not only obvious but acceptable rationalizations and explanations. At a later date somebody may dig out this book from its time capsule and marvel about the 1960–1970 period and the wonders of self-transcendence, that is the capability of a system to represent itself.

Kierkegaard described the self as a "relation which is related to itself". And Cocteau, in "The Infernal Machine" exclaimed, "Could my brain be an enchanted mirror?" Both men point to two dimensions of the mental structure of consciousness, the *denotative* ('being conscious of other things') and a *meta*-dimension ('being conscious of oneself'). The self-reflection of 'observer in the observed' is implicit in the unique complementarity of having and being a body (of knowledge). Or, as Valéry puts it, "The self is in the world which is in the self" or even more explicitly "I am *in* a world which is *in* me." In English 'conscious' as meaning 'inwardly sensible or aware' appeared first in 1620, 'consciousness' or 'the state of being conscious' in 1678, and 'self-consciousness' or 'consciousness of one's own thoughts', in 1690. It is also interesting, according to Lancelot Law Whyte, that 'con-scious' whose Latin source had meant 'to know with' (to share knowledge with another) now came to mean 'to know in oneself, alone'. It took two hundred more years of loneliness and individuation for another new word to be born, 'psychotherapy'.

*Scio* in con-scientia of course means to cut or to make distinctions; hence knowing-being refers (1) to consensually validated con-sciousness (with others), (2) to being conscious of oneself and (3) to one's ability to make distinctions. Erik Erikson would also remind us of another connotation, self-conscious, meaning that one is completely exposed and conscious of being looked at (like this book reviewer, now).

It becomes obvious—after reading the book—that the appearance of living structures (on earth) is implicit and thus a 'way of life' of the Second Law of Thermodynamics. Or in other words, it is a characteristic of matter-energy to pay a price for becoming alive and conscious of itself. That price is called, collectively, heat-death. Or in Borges' words: "Time is a river which sweeps me along, but I am the river; it is a tiger which destroys me, but I am the tiger; it is a fire which consumes me, but I am the fire".

Just as the mechanization of the universe was completed in 1619 by Newton, so the transferring of God's transcendental 'functions' to matter (-energy) has been completed in our days by Prigogine. His elevation to "sainthood" in the form of a Nobel Prize is the well deserved validation of an already established belief-system. Like God, the creator of man in his own image, matter has now become its own self-referential creator. Apparently, the Genesis in the Old Testament and the Second Law re-present the very same medium; hence, in the beginning *was* the message.

The book is divided in three parts: Part I is written by Waddington, Pankow and Jantsch and culminates in the evolutionary concept of self-realization through self-transcendence. Pankow's scientific dissection of science is in itself a masterful example of self-transcendence:

> ...To the extent that science becomes tautological without knowing it and therefore insists on its claim of explaining the outer world, it will try to transform the outer world according to its own image and in this way it will become a self-fulfilling prophecy.

Part II deals with Continuity in Change and contains a chapter by Holling who develops the concept of multiple stable regions in phase space, a boundary-oriented view of ecosystems. Evolution does not appear to be a capitalistic process; it rather maximizes flexibility, not efficiency nor productivity, to persist. Then Prigogine presents his dynamic concept of "order through fluctuation" and life as an adaptable state of metastability. Abrams discusses morphogenesis in the light of Thom and Zeeman's Catastrophe Theory, and in the last chapter of Part II Zeleny and Pierre deal with the simulation of self-renewing systems (such as cells) with a view to their potential application to human system management.

In Part III, Taylor, Marney and Schmidt, Maruyama, Markley and Jantsch apply, in their own original way, Prigoginean concepts to sociocultural system, noetic evolution, a mutual-causal logic, a coming image of man and the integration (re-ligio) and differentiation (history) in three waves, i.e., grounding, socialization and individuation.

The book has a nostalgic-utopian touch. It may not be an overstatement that it constitutes the scientific foundations of 'hope in transition'. It often displays religious overtones though always couched in cybernetic, systems theoretical or other scientific terminology. Jantsch's quest for absolute values becomes an integral aspect of consciousness and its evolution forming the core of an internal – superconscious – learning process of mankind. Second *Law and "order* through fluctuation"and the oceanic feeling of being part of an infinite process of cosmological dimensions are the impetus to mold the future of that universe which is ourselves. This is an enlightening book. Not that, after reading it, we can see the light at the end of the tunnel ... rather we receive light and enlightenment for tunnel building; and do so in both of our cerebral hemispheres. This book is highly recommended avant-garde reading.

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LA THEORIE DU SYSTEME GENERAL: THEORIE DE LA MODELISATION (GENERAL SYSTEMS THEORY: THEORY OF MODELLING), by Jean-Louis Le Moigne, Presses Universitaires de France, Paris, 1977. 258 pp.

This is a good book. If I had to teach an introductory systems course, I would choose it as a core reference. Here is why I think this is the case.

The first reason is that this book aims at the middle ground between two major extremes which I believe are fatal. One extreme is to see systems theory as a transdisciplinary language, where isomorphisms play the major role. The other extreme is systems theory as if there was nothing else except engineering modelling. The middle road is to see systems theory as a contemporary cosmology of sorts, not more, not less. Parallel to what the theory of matter was for the 18th century. Their natural philosophy was concerned with matter and quantity; our natural philosophy seems more concerned with relation and process (cf. Bunge, M., 1977 in this Journal). I have yet to see a recent book in English that does not deviate substantially to one or the other extreme.

This book does get closer to that middle ground of an up-to-date panorama of the technical details and applications, yet with a certain sense of aerial perspective for conceptual and foundational issues. I suspect this is because M. Le Moigne is quite close to the technological details (he is Professor of Systems Analysis in Marseille, and was for many years in industry and management), but he is also a Frenchman with an ear for the current discussions on foundational problems. Thus he achieves a certain degree of transcontinentality which is refreshing. Much of the contribution from Eastern Europe is, alas, poorly represented.

The book is organized according to a simple principle. He devotes the first three chapters to discuss the "Systems Paradigm", concluding with a chart of how the systems-object can be analyzed in a two dimensional conceptual matrix. In the columns we find the system defined in its functional, ontological, and genetic dimensions. In the row one considers the system considered as activity, structure, and evolution. This gives a total of 9 loci of interdependency, confirming the nine chapters of the book. To be sure this is not the only way of organizing a presentation, and there is some evident overlap, but it does provide an interesting guiding thread.

The book's greatest weakness, in my view, is that it contains little critical evaluation of the work contained and reported in it. It reads more like a survey. This makes it very difficult for a student to have a least one opinion as to what are the central results and ideas in the confusing jungle we are bent on calling systems theory. A second weakness is that the material is uniformly presented at a superficial level. At no point we are taken to the guts of a particular problem, model, or perspective. Granted that this could not be done for all topics mentioned; but one or two cases would make the reading less fuzzy. Fortunately a good bibliography does provide entry points to the relevant literature.

These are weakness which are far outweighed by the quality of this book. It should be read by students and researchers alike.

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SYSTEM RELIABILITY MODELING AND EVALUATION, by Chanan Singh and Roy Billinton, Hutchinson, London, 1977. 248 pp.

This book is concerned with mathematical models of system reliability and with the methodology through which it is possible to analyze systems reliability under various conditions. The book defines a series of reliability characteristics on the basis of various statistical information. In particular, systems failures and repair cycles of the systems constituent components are used to determine different reliability indicators, both in maintained and non-maintained systems.

The book is divided into eight chapters. The first two chapters survey basic concepts of probability theory and stochastic processes. In these chapters emphasis is placed upon concepts of mathematical statistics that can be used for constructing various systems models and for designing reliability evaluation procedures. The third chapter deals with the concept of frequency, renewal theory and with the concepts of transition rates. The fourth chapter discusses the concepts of systems modeling within the context of systems reliability and describes the approaches that can be used for reliability assessment. The fifth chapter introduces analytical techniques that can be used for reliability evaluation of relatively large and complex systems, both non-repairable and repairable. The sixth chapter deals with the reliability modeling in non-Markovian systems and discusses semi-Markovian processes. The seventh chapter surveys some of the elementary simulation concepts, that can be used to estimate various reliability measures. The final chapter briefly summarizes the objectives of the book.

The chapters are well written and well documented. Each chapter has attached list of references, pertinent to the subject matter. The text contains a great deal of mathematical formalization and therefore, it would be of the greatest benefit to those, who already have a deeper knowledge of mathematical statistics, especially a knowledge of stochastic processes and probability theory. Since the book is not aimed at any specific area of application and since it deals with the systems reliability in general terms, the book can be recommended as a useful source of information to all those systems specialists and generalists who are interested in the analysis of systems reliability in their particular area of endeavor and who are interested in the design and construction of more reliable and more effective systems.

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DYNAMICAL SYSTEMS AND THEIR APPLICATIONS: LINEAR THEORY, by John L. Casti, Academic Press, New York, 1977. xv + 240 pp.

Really this book presents the theory of linear dynamic systems rather than a complete 'linear theory' of dynamic systems. There is a difference in that a book on linear theory would contain the development of a consistent mathematical theory starting from pre-ordained axioms. Though the present volume contains aspects of this its main pre-occupation is with the conventional presentation of a large number of diverse, well-known control results proved using linear algebra. Section 4.5 on page 81 on a functional analytic approach to observability is consequently an anomaly.

In the preface the author makes much of the need to know solid mathematical systems theory in order to make meaningful inroads into the pressing problems which confront our modern society—take heed those lecturers and students who wittingly or unwittingly are encouraging Mickey Mouse systems analysis courses in the name of relevance. However the author then lays himself open to criticism by peppering his chapters with superficial models of 'real world' problems.

In Chapter 1—basic concepts, problems and examples—the author introduces the notion of a linear dynamic system and of the various properties which such systems can exhibit. In the example on industrial production a lack of care leads to the use of the terms output rate and output for the same variable. I could not get Figure 1.2 to agree with the equations which follow it; generally I found the figures misleading, uninformative and drawn using non-standard conventions. On page 6 the author states that given any external description of $f$ it is always possible to find an internal realization. However an infinite-dimensional realization may be required and this should have been mentioned as the preceding pages deal only with finite-

dimensional differential equations. On page 9 there is confusion between the output at time $t, y(t)$, and the output function, $y(\tau), \tau \in [0, t]$. Inclusion of the term $(x(T), Mx(T))$ in the quadratic functional $J$ makes sense even when $T \to \infty$, so that the restriction $T < \infty$ on page 14 is overly severe.

Chapter 2—mathematical description of linear dynamical systems—is concerned mainly with definitions. There are also sketchy sections on input–output descriptions and transfer functions. On page 22 $u$ is used to represent the function $u(\cdot)$ while elsewhere, e.g. Eq. (2.1), $u$ means the value of the control at time $t$, viz. $u(t)$. This inconsistency is present also in Chapter 3 and actually throughout the book.

Chapter 3—controllability and reachability—is fairly standard with only a few novel insights. On page 39 it should have been emphasized that $U \equiv R^m$. The section on positive controllability is sketchy and it is a pity that Brammer's elegant results are not emphasized. The sections on relative controllability and conditional controllability are useful, though straightforward.

Chapter 4—observability and constructability—is brief and standard.

Chapter 5—structure theorems and canonical forms—is also standard but it is good to have results in this area collected together. However, there is no reference to Luenberger's papers on canonical forms, which is an omission.

Chapter 6—realization theory—is standard and brief.

Chapter 7—stability theory—contains no surprises and is a reasonable introduction to stability. The uniqueness statement in the pole-shifting theorem on page 159 is not valid when the control is an $m$-vector.

Chapter 8—linear-quadratic-Gaussian problems—contains two sections (Sections 8.6 and 8.8) on the author's speciality which are surprisingly brief. Section 8.9 on computational approaches for Riccati equations is overly skimpy when viewed in the light of the myriad papers on this subject that have appeared during the past 10 years. Section 8.12 on continuous time filtering should have been preceded by Section 8.14 on the discrete-time formulation. One could then have 'waved hands' to explain unrigorously continuous time Gaussian white noise. As it stands, Section 8.12 is misleading in that it gives the impression that the differential equation with noise as a forcing function is a simple extension of the deterministic case and does not raise any new theoretical questions of existence and so on.

Viewed as a whole the book is yet another devoted to (a segment of) linear control theory. In parts it is too sketchy and the references to the literature are far too incomplete. The only sections on recent extensions of standard results are Section 3.6 on positive controllability and the author's work on low-dimensional approaches to the Riccati equation in Sections 8.6 and 8.8. The index is not extensive and the exercises do not appear to be original.

Students and lecturers may find this book useful, but they will not feel that it is the deep scholarly work which the knowledgeable author undoubtedly intended it to be.

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# *Current Literature*

Alspach, B., P. Hell and D. J. Miller (editors), *Algorithmic Aspects of Combinatorics.* North-Holland, New York, 1978, VI + 245 pp.

Anderton, R., "Systems in the seventies: An emerging discipline?" *J. of Applied Systems Analysis*, **5,** No. 2, May 1978, pp. 149–156.

Andrews, J. G. and R. R. McLone (editors), *Mathematical Modelling.* Butterworths, London, 1976, 260 pp.

Baase, S., *Computer Algorithms.* Addison-Wesley, Reading, Mass., 1978, 286 pp.

Balkus, K. (editor), *Systems in Sociocultural Development.* Proc. of SGSR 1978 Southeastern Meeting, Tallahassee, Florida, 1978.

Batchelor, B. G. (editor), *Pattern Recognition: Ideas in Practice.* Plenum Press, New York, 1978.

Bathe, K. J., *et al.* (editors), *Formulations and Computational Algorithms in Finite Element Analysis: U.S.-German Symposium.* M.I.T. Press, London, 1977, 1091 pp.

Battista, J. R., "The holistic paradigm and general systems theory." *General Systems Yearbook*, **22,** 1977, pp. 73–83.

Bender, E. A., *An Introduction to Mathematical Modelling.* John Wiley, New York, 1978, 256 pp.

Bezdek, J. C. and J. D. Harris, "Fuzzy partitions and relations: An axiomatic basis for clustering." *Fuzzy Sets and Systems* **1,** No. 2, April 1978, pp. 111–127.

Blauberg, I. V., V. N. Sadovsky and E. G. Yudin, *Systems Theory: Philosophical and Methodological Problems.* Progress Publishers, Moscow, 1977.

Botez, M. C., "Toward a systemic representation of general systems: Open multimodelling." *Modern Trends in Cybernetics and Systems*, edited by J. Rose, Springer-Verlag, New York, 1977.

Branin, F. H. and K. Husseyin (editors), *Problem Analysis in Science and Engineering.* Academic Press, New York, 1977, 514 pp.

Bridge, J., *Beginning Model Theory.* Clarendon Press, Oxford, 1977, 143 pp.

Broekstra, G., "Constraint analysis and structure identification II." *Annals of Systems Research*, **6,** 1977, pp. 1–20.

Bubnicki, Z., "Global and local identification of complex systems with cascade structure." *Systems Science*, **1,** No. 1, 1975, pp. 55–65.

Bunge, M., "General systems and holism." *General Systems Yearbook*, **22,** 1977, pp. 87–90.

Burns, J. R. and O. Ulgen, "A sector approach to the formulation of systems dynamics." *Int. J. of Systems Science*, **9,** No. 6, 1978, pp. 649–680.

Cavallo, R. (editor), *The Role of Systems Methodology in Social Science Research.* Martinus Nijhoff, Boston, 1978. (Volume 1 in Book Series on *Frontiers in Systems Research: Implications for the Social Sciences*).

Cavallo, R. (editor), *Recent Developments in Systems Methodology for Social Science Research.* Martinus Nijhoff, Boston 1979, pp. (Volume 2 in Book Series on *Frontiers in Systems Research: Implications for the Social Sciences*).

Chaitin, G. J., "Algorithmic information theory." *IBM Journal of Research and Development*, **21,** No. 4, July 1977, pp. 350–359.

Cremers, A. B. and T. N. Hibbard, "Orthogonality of information structures." *Acta Informatica*, **9,** No. 3, 1978, pp. 243–261.

Checkland, P. B., "The origins and nature of 'hard' systems thinking." *J. of Applied Systems Analysis*, **5,** No. 2, May 1978, pp. 99–110.

DeGreene, K. B., "Force fields and emergent phenomena in socio-technical macrosystems: Theories and models." *Behavioral Science*, **23,** No. 1, January 1978, pp. 1–14.

Denning, P. J., J. B. Dennis and J. E. Qualitz, *Machines, Languages and Computation.* Prentice-Hall, Englewood Cliffs, N.J., 1978, 601 pp.

Dijkstra, W. and J. van der Zouwen, "Testing auxiliary hypothesis behind the interview." *Annals of Systems Research*, **6,** 1977, pp. 49–63.

Dreyfus, S. E., *Dynamic Programming and the Calculus of Variations.* Academic Press, New York, 1965, 248 pp.

Frank, P. M., *Introduction to System Sensitivity Theory.* Academic Press, New York, 1978, 400 pp.

*Frontiers in Systems Research: Implications for the Social Science*, A new book series, edited by George J. Klir, Stein Braten and John Casti; Martinus Nijhoff, 160 Old Derby Street, Hingham, Mass. 02043, U.S.A., or Martinus Nijhoff, P. O. Box 442, The Hague, The Netherlands (see R. Cavallo in this list).

Genesio, R. and N. Sorrentino, "On the model order for uncertain system approximation." *Systems Science*, **3,** No. 3, 1977, pp. 215 225.

Ghosal, A., *Applied Cybernetics: Its Relevance to Operations Research.* Gordon and Breach, New York, 1978, 180 pp.

Gold, H. J., *Mathematical Modeling of Biological Systems: An Introductory Guidebook.* John Wiley, New York, 1977, 357 pp.

Golemanov, L. A. and L. Hakkala, "On the coordinability of hierarchical systems." *Systems Science*, **3,** No. 3, 1977, pp 285–297.

Gonzales, R. C. and P. Wintz, *Digital Image Processing.* Addison-Wesley, Reading, Mass., 1977.

Gorbatov, V. A., *Theory of Partially Ordered Systems* (in Russian). Sovietskoye Radio, Moscow, 1976, 336 pp.

Gottinger, H. W., "Toward an algebraic theory of complexity of dynamic systems." *General Systems Yearbook*, **22,** 1977, pp. 73–83.

Gottinger, H. W., "Complexity and catastrophe: Applications of dynamic system theory." *Modern Trends in Cybernetics and Systems*, Vol. II, edited by J. Rose and C. Bilciu, Springer-Verlag, New York, 1977, pp. 13 26.

Gyula, P., *Common Laws of Sciences and Systems* (in Hungarian). Ahadémiai Kiadó, Budapest, 1973.

Hägg, C., "Possibility and cost in decision analysis." *Fuzzy Sets and Systems*, **1,** No. 2, April 1978, pp. 81–86.

Haimes, Y. Y., *Hierarchical Analysis of Water Resources Systems: Modeling and Optimization of Large-Scale Systems.* McGraw-Hill, New York, 1977, 478 pp.

Hájek, P. and T. Havránek, *Mechanizing Hypothesis Formation—Mathematical Foundations of a General Theory*, Springer-Verlag, New York, 1978.

Haralik, R. M., L. S. Davis, A. Rosenfeld and D. L. Milgram, "Reduction operation for constraint satisfaction." *Information Sciences*, **14,** No. 3, June 1978, pp. 199 219.

Harrison, M. A., *Introduction to Formal Language Theory.* Addison-Wesley, Reading, Mass., 1978, 594 pp.

Hazlett, B. A. (editor), *Quantitative Methods in the Study of Animal Behavior.* Academic Press, New York, 1977, X + 222 pp.

IEEE Proc., May 1979, *Special Issue on Pattern Recognition.*

IEEE Trans. on Circuits and Systems, Sept. 1978, *Special Issue on the Mathematical Foundations of System Theory.*

Ingermarsson, I., "Toward a theory of unknown functions." *IEEE Trans. on Information Theory*, **IT-24,** No. 2, March 1978, pp. 238–240.

Jacak, W. and K. Tchón, "Events and systems." *Systems Science*, **1,** No. 2, 1975, pp. 77–88.

Jacak, W. and K. Tchón, "On goal-oriented systems." *System Science*, **3,** No. 3, 1977, pp. 307–313.

Jakubowski, R. and J. Król, "General algorithms of complex dynamic systems simulation." *Systems Science*, **1,** No. 1, 1975, pp. 17 27.

Jaroń, J., "Goal-oriented cybernetical systems." *Systems Science*, **1,** No. 1, 1975, pp. 5–16.

Jaroń, J., "Relations and topologies in the goals-space of a system." *Systems Science*, **3,** No. 2, 1977, pp. 101–118.

Ježek, J., *Universal Algebra and Theory of Models* (in Czech). SNTL, Prague, 1976, 226 pp.

Kaufmann, A., D. Grouchko and R. Cruon, *Mathematical Models for the Study of the Reliability of Systems.* Academic Press, New York, 1977, 221 pp.

Keen, P. G. W. and M. S. S. Morton, *Decision Support Systems: An Organizational Perspective.* Addison-Wesley, Reading, Mass., 1978, 264 pp.

Kilmann, R. H., *Social Systems Design: Normative Theory and the MAPS Technology.* North-Holland, New York, 1977, XV + 327 pp.

Klir, G. J., "The general systems research movement." *Systems Models for Decision Making*, edited by N. Sharif and P. Adulbhan, Asian Institute of Technology, Bangkok, 1978, pp. 23–70.

Koppelaar, H. and D. Kruijt, "Regular career systems II." *Annals of System Research*, **6,** 1977, pp. 35–47.

Lamport, L., "Time, clock and the ordering of events in distributed systems." *ACM Communications*, **21,** No. 7, July 1976, pp. 558–565.

Langefors, B. and K. Samuelson, *Information and Data in Systems.* Petrocelli, New York, 1976.

Laszlo, E., "GST: Prospects and principles." In: *Modern Trends in Cybernetics and Systems*, Vol. 1, pp. 131–152, edited by J. Rose and C. Bilciu, Springer-Verlag, New York, 1977.

Leung-Yan-Cheung, S. K. and T. M. Cover, "Some equivalences between Shannon entropy and Kolmogorov complexity." *IEEE Trans. on Information Theory*, **IT-24,** No. 3, May, 1978, pp. 331–338.

Lilienfeld, R., *The Rise of Systems Theory: An Ideological Analysis.* John Wiley, New York, 1978.

Mamdani, E. H., "Application of fuzzy logic to approximate reasoning using linguistic synthesis." *IEEE Trans. on Computers*, **C-26,** No. 12, December 1977, pp. 1182–1191.

Meldman, M. J. *et al.*, *RISS: A Relational Data Base Management System for Minicomputers.* Van Nostrand Reinhold, New York, 1978, IX + 113 pp.

Metz, H. A. J., "State space models for animal behaviour." *Annals of Systems Research*, **6,** 1977, pp. 65–109.

Metzner, J. R. and B. H. Barnes, *Decision Table Languages and Systems.* Academic Press, New York, 1977.

Miller, J. G., *Living Systems.* McGraw-Hill, New York, 1978, 1102 + xii pp.

Moylan, P. J. and D. J. Hill, "Stability Criteria for large-scale systems." *IEEE Trans. on Automatic Control*, **AC-23,** No. 2, April 1978, pp. 143–149.

Nahmias, S., "Fuzzy variables." *Fuzzy Sets and Systems*, **1,** No. 2, April 1978, pp. 97–110.

Netchiporenko, V. I., *Structure Analysis of Systems* (in Russian). Sovietskoye Radio, Moscow, 1977, 214 pp.

Nicolis, J. S. and M. Benrubi, "Inadequate communication between self-organizing systems and desynchronization of physiological rhythms." *General Systems Yearbook*, **22,** 1977, pp. 119–136.

Nurmi, H., "On strategies of cybernetic model-building." *Kybernetes*, **7,** No. 1, 1978, pp. 13–18.

O'Muircheartaigh, C. A. and C. Payne (editors), *The Analysis of Survey Data, Vol. 1: Exploring Data Structures. Vol. 2: Model Fitting.* John Wiley, New York, 1977.

Otnes, R. K. and L. Enochson, *Applied Time Series Analysis.* Volume I: *Basic Techniques.* John Wiley, 1978, XIV + 449 pp.

Pearl, J., "An economic basis for certain methods of evaluating probabilistic forecasts." *International Journal of Man-Machine Studies*, **10,** No. 2, March 1978, pp. 175–183.

*Policy Analysis and Information Systems.* A new journal published semi-annually by the University of Illinois at Chicago Circle and Tankang College in Taipei, Taiwan. Editor-in-Chief: Shi-Kuo Chang, Dept. of Information Engineering, University of Illinois at Chicago Circle, Ill. 60680, U.S.A. The journal has been published since 1977.

*Postepy Cybernetyki* (*Advances in Cybernetics*). A new quarterly published by the Polish Cybernetics Society (in Polish).

Poston, T. and I. Stewart, *Catastrophe Theory and its Applications.* Pitman, London, 1978, 491 pp.

Pratt, W. K., *Digital Image Processing.* John Wiley, New York, 1978, X + 750 pp.

Rastogi, P. N., *The Behaviour of Societal Systems.* Indian Institute of Advanced Study, Simla, India, 1978.

Rescher, N., *Methodological Pragmatism: A Systems-Theoretical Approach to the Theory of Knowledge.* New York Univ. Press, New York, 1977, 315 pp.

Reuver, H. A., "Learning within the context of general systems theory." *Progress in Cybernetics and Systems Research*, Vol. IV, edited by R. Trappl and G. Pask, Hemisphere, Washington, D.C., 1978, pp. 113–121.

Rose, J. (editor), *Modern Trends in Cybernetics and Systems.* Springer-Verlag, New York, 1977.

Rosen, R., *Findamentals of Measurement and Representation of Natural Systems.* North-Holland, New York, 1978, XVII + 221 pp. (Volume 1 in *The North-Holland Series in General Systems Research.*)

Rosenberg, R. M., *Analytical Dynamics of Discrete Systems.* Plenum Press, New York, 1977, 424 pp.

Rosenkrantz, R. D., *Inference, Method and Decision.* D. Reidel, Dordrecht-Holland and Boston, 1977, 262 pp.

Saeks, R. (editor), *Large-Scale Dynamical Systems.* Western Periodicals, North Hollywood, Ca., 1976, 314 pp.

Sagal, P. T., "Epistemology of economics." *Journal for General Philosophy of Science*, **8,** No. 1, 1977, pp. 144–162.

Sage, A. P., *Methodology for Large Scale Systems.* McGraw-Hill, New York, 1977.

Sahal, D., "Towards a Theory of Systems." *General Systems Yearbook*, **22,** 1977, pp. 41–55.

Sahal, D., "Principles of regulation and control." *Kybernetes*, **7,** No. 1, 1978, pp. 19–24.

Samuelson, K., H. Borko and G. X. Amey, *Information Systems and Networks.* North-Holland, New York, 1977, X + 148 pp.

Saridis, G. N., *Self-Organizing Control of Stochastic Systems.* Marcel Dekker, New York, 1977, 488 pp.

Schilderinck, J. H. F., *Regression and Factor Analysis in Econometrics.* Martinus Nijhoff, Leiden, 1977, 239 pp.

Sharif, N. and P. Adulbhan (editors), *Systems Models for Decision Making.* Asian Institute of Technology, Bangkok, 1978, 433 pp.

Sharif, N. and P. Adulbhan (editors), *Proceedings of the International Conference on Systems Modelling in Developing Countries.* Asian Institute of Technology, Bangkok, 1978.

Shieh, L., R. E. Yates and J. M. Navarro, "Representation of continuous time state equations by discrete-time state equations." *IEEE Trans. on Systems, Man and Cybernetics*, **SMC-8,** No. 6, June 1978, pp. 485–492.

Šiljak, D. D., *Large-Scale Dynamic Systems.* North-Holland, New York, 1978, xvi + 416 pp.

Silvern, L. C., *Systems Engineering of Education I: The Evolution of Systems Thinking in Education.* ETC, Los Angeles, 1975 (Third Edition), 128 pp.

Silviu, G., *Information Theory with Applications.* McGraw-Hill, New York, 1977, 439 pp.

Simon, H. A., *Models of Discovery and Other Topics in the Methods of Science.* D. Reidel, Boston, 1977, XX + 464 pp.

*Simulation Newsletter*, edited by David Ellison and published intermittently by the Centre in Simulation, Gillow House, The University of Lancaster, Lancaster LA1 4YG, England.

Skala, H. J., "On many-valued logics, fuzzy sets, fuzzy logics and their applications." *Fuzzy Sets and Systems*, **1,** No. 2, April 1978, pp. 129–149.

*Special Issue of the International Journal of Man-Machine Studies on the GUHA Method of Mechanized Hypothesis Formation*, **10,** No. 1, January 1978, pp. 1–93. Guest Editor: P. Hájek.

Special Issue on Learning Automata, *Journal of Cybernetics and Information Science*, **1,** No. 1–4, 1977.

Stegmüller, W., *The Structure and Dynamics of Theories.* Springer-Verlag, 1976.

Stone, L. D., *Theory of Optimal Search.* Academic Press, New York, 1975.

Stoy, J. E., *Denotational Semantics: The Scott–Strachey Approach to Programming Language Theory.* The M.I.T. Press, Cambridge, Mass., 1977, 414 pp.

Sussmann, H. J. and R. S. Zahler, "Catastrophe theory as applied to the social and biological sciences: a critique." *Synthese*, **37,** No. 2, February 1978, pp. 117–216.

Sutherland, J. W., *Societal Systems Methodology, Modelling and Management.* North-Holland, New York, 1978, XV + 336 pp.

*Systems Science.* A journal published quarterly by Wroctaw Technical University (Poland) since 1975. *Contact*: Prof. Z. Bubnicki, Editor, Wroclaw Technical University, Institute of Engineering Cybernetics, Wroclaw, Janiszewskiego 11/17, Poland.

Takahara, Y. and H. Ikeshoji, "Characterization of the causality of general dynamic systems." *Int. J. of Systems Science*, **9,** No. 6, 1978, pp. 639–648.

*The North-Holland Series in General Systems Research*, edited by George Klir; Elsevier North-Holland, Inc., 52 Vanderbilt Avenue, New York, N.Y. 10017, U.S.A., or Thomond Books, P.O. Box 85, Limerick, Ireland (see R. Rosen in this list).

Tapiero, C. S., *Managerial Planning: An Optimum and Stochastic Control Approach* (2 Volumes). Gordon and Breach, New York, 642 pp.

Thissen, W., "Investigations into world 3 model: lessons for understanding complicated models." *IEEE Trans. on Systems, Man, and Cybernetics*, **SMC-8,** No. 3, March 1978, pp. 183–193.

Toffoli, T., "Computation and construction universality of reversible cellular automata." *Journal of Computer and Systems Sciences*," **15,** 1977, pp. 213-231.

Topolski, J., *Methodology of History*. D. Reidel, Dordrecht and Boston, 1976, X + 690 pp.

Trappl, R., *et al.* (editors), *Progress in Cybernetics and Systems Research* (Volumes 3 and 4). Hemisphere, Washington, D.C., 1978.

Vemuri, V., *Modeling of Complex Systems: An Introduction*. Academic Press, New York, 1978, 464 pp.

Windeknecht, T. G. and H. D'Angelo, "System theoretic implications of numerical methods applied to the solution of ordinary differential equations." *IEEE Trans. on Systems, Man and Cybernetics*, **SMC-7,** No. 11, 1977, pp. 805–810.

Yager, R. R., "Fuzzy decision making including unequal objectives." *Fuzzy Sets and Systems*, **1,** No. 2, April 1978, pp. 87–95.

Yeh, R. T. (editor), *Current Trends in Programming Methodology: Volume IV, Data Structuring*. Prentice-Hall, Englewood Cliffs, N.J., 1978, 321 pp.

Yezhkova, I. V. and D. A. Pospelov, "Decision making on fuzzy grounds." *Engineering Cybernetics*, 1977, No. 6.

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**010477**
A SYSTEMS INTERPRETATION OF MULTIDISCIPLINARY AND INTERDISCIPLINARY INQUIRY IN CURRICULUM DESIGN, by Myrna L. Estep, *University of Texas at San Antonio*, 20 *pp.* ($1.50).

As the terms "multidisciplinary" and "interdisciplinary" are frequently used in general systems literature without a clear determination of their meanings, I sought to show a logical and epistemological interpretation of their meanings in the context of an organismic model for curriculum design. Logical and epistemological categories of knowledge and knowing were utilized to illustrate the possibilities for multidisciplinary and interdisciplinary inquiry, and to illustrate an organismic theory models approach to educational inquiry. This approach necessitates that the teaching-learning process be viewed as a structured whole, i.e. one in which the content and form of its parts are determined by its function, thus not having non-alterable natures and fixed actions or relations.

**120577.1**
MINIMIZING WAITING TIMES USING PRIORITY CLASSES: A CASE STUDY IN RESPONSE SURFACE METHODOLOGY, by W. van den Bogaard and J. P. C. Kleinen, *Department of Business and Economics, Katholieke Hogeschool, Tilburg, The Netherlands*, ($1.00).

Key-punching in our computer center is modelled as a queuing simulation with two servers (typists) and three priority classes (small, medium, large jobs). The 90% quantiles of the waiting times per job class are estimated for different borderlines ($X$ and $Y$) between the three job classes. We investigate several regression equations, expressing the quantiles as functions in $X$ and $Y$. Response Surface Methodology (a hill-climbing method) is applied to find the optimal class limits. This leads to a reformulation of the problem, explicitly incorporating trade-offs among the quantiles of each of the three job classes. Approximately optimal class limits are determined. We demonstrate that reducing the number of job classes from three to two gives bad results.

**060878.1**
INVENTORY OF SYSTEMS RESEARCH IN THE NETHERLANDS, by Johannes Van der Zowen, *Vrije Universiteit, Amsterdam, The Netherlands*, 15 pp. ($1.00).

This is an updated version of the description of systems research in the Netherlands which was published in Volume 4, No. 1, pp. 75–79, of this journal.

**091478.1**
ON SYSTEMS WITH REDUNDANCY AND THEIR INHERENT BOUNDS ON ACHIEVABLE RELIABILITY, by Peter W. Becker, *Technical University of Denmark, Electronics Laboratory, Lyngby, Denmark*, 7 pp. ($1.00).

Consider a system which is supposed to bring some commodity (e.g. electric power, gas, oil, telephone messages) from one point $P$ to another point $T$. To avoid total disruption of the services several parallel connections between $P$ and $T$ may be built, i.e. the designer tries to enhance the system's reliability through use of redundancy. In the paper a method, the so-called $\theta$-transformation, is described by which the highest and lowest system reliability achievable may be determined for a redundant configuration. As a by-product we become able to pinpoint the statistical relationships which give rise to highest and lowest system reliabilities.

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# *Calendar*

**August 6–17, 1979**
FIRST INTERNATIONAL SUMMER SCHOOL ON SYSTEMS METHODOLOGY
Delft, The Netherlands. *Contact* G. Broekstra, Graduate School of Management, Poortweg 6–8, Delft, The Netherlands.

**August 20–24, 1979**
SIXTH INTERNATIONAL JOINT CONFERENCE ON ARTIFICIAL INTELLIGENCE
Tokyo, Japan. *Contact* Raj Reddy, Dept. of Computer Science, Carnegie-Mellon University, Schenley Park, Pittsburgh, Pennsylvania 15213, U.S.A.

**August 21–24, 1979**
INTERNATIONAL MEETING OF THE SOCIETY FOR GENERAL SYSTEMS RESEARCH
London, England. *Contact* Richard Ericson, George Washington University, Lisner Hall, 2023 G. Street N.W., Washington D.C. 20006, U.S.A.

**August 22–29, 1979**
SIXTH INTERNATIONAL CONGRESS OF LOGIC, METHODOLOGY AND PHILOSOPHY OF SCIENCE
Hanover, Federal Republic of Germany, *Contact* Secretariat des Internationalen Wissenschatten Welfengarten 1, D-3000 Hanover, BRD.

**September 10–14, 1979**
NINTH INTERNATIONAL CYBERNETICS CONGRESS
Namur, Belgium. *Contact* J. Lemaire, Managing Director, IAC, Palais des Expositions, Place Andre' Rijckmans, B-5000, Namur, Belgium.

**October 8–12, 1979**
INTERNATIONAL SYMPOSIUM ON SYSTEM-MODELING-CONTROL
Zakopane, Poland. *Contact* Center of Electronic Computing, Technical University of Lodz, Ul. Piotrkowska 220, 90–369 Łodž, Poland.

**January 4–5, 1980**
THE TWELFTH HAWAII INTERNATIONAL CONFERENCE ON SYSTEMS SCIENCES
Ilikai Hotel, Honolulu, Hawaii. *Contact* Ralph H. Sprague, Jr., College of Business Administration, University of Hawaii, 2404 Maile Way, Honolulu, Hawaii 96822, U.S.A.

**January 7–10, 1980**
TWENTY-FOURTH ANNUAL MEETING OF THE SOCIETY FOR GENERAL SYSTEMS RESEARCH
San Francisco, California. *Contact* Bela H. Banathy, Far West Laboratory for Educational Research and Development, 1855 Folson Street, San Francisco, California 94103 U.S.A.

**April 8–11, 1980**
FIFTH EUROPEAN MEETING ON CYBERNETICS AND SYSTEMS RESEARCH
Vienna, Austria. *Contact* Robert Trappl, Österreichische studiengesellschaft fur Kybernatik, Schottengasse 3, A-1010 Wien 1, Austria. The Conference will include a Symposium on General Systems Methodology. Papers submitted to this Symposium should be sent to George J. Klir, Dept. of Systems Science, SAT, SUNY, Binghamton, New York 13901, U.S.A.

# DIGITAL PROCESSES

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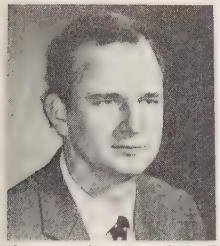

**Carl Albrecht** was born in Johannesburg in 1943. In 1964, he received the degree of B.Sc.(Hon.) in Biochemistry from the University of the Witwatersrand. In 1970 he received a Ph.D. in Biochemistry from the same university. His thesis work was titled—"The isolation, fractionation and sub-fractionation of rat liver nuclei, applied to studies of azo dye carcinogenesis". From 1966 to 1974, he was a research scientist in the Biochemistry Division of the National Chemical Research Laboratory of the Council for Scientific and Industrial Research, engaging in molecular biological research which is aimed at understanding the nature of gene-control in normal and cancer cells. Since 1975, he has been a Chief Professional Officer in the Department of Pharmacology of the Stellenbosch Medical School, where he teaches basic mechanims of drug action and does research on proteins released by cancer cells in tissue culture.

Carl Albrecht's statement:

*There is no doubt in my mind that general systems theory will play a very important role in the elucidation of the "big" problems in biology, such as: gene control, embriogenesis, how the mind works, the nature of the cancer cell and the process whereby life evolved on this planet. We have only scratched the surface and many biologists are realizing that we must move beyond reductionism towards synthesis—in order to understand the phenomena of life. To this end I am indeed priviledged to be associated with the International Journal of General Systems.*

**Peter W. Becker** received his M.Sc. and Dr.Techn. degree in Electrical Engineering from the Technical University of Denmark in 1950 and 1968, respectively. Since 1967 he has been a member of the staff at the Electronics Laboratory of the Technical University of Denmark, where he teaches Reliability Theory and Pattern Recognition Theory.

Dr. Becker's previous technical experience includes an eight year period, 1958 to 1966, with the General Electric Co., Electronics Laboratory, Syracuse, N.Y., preceded by two years with other G.E. laboratories.

Dr. Becker has written several papers in the field of reliability. In 1977, he published the book "Design of Systems and Circuits for Maximum Reliability or Maximum Production Yield" (co-authored by Mr. Finn Jensen) with McGraw-Hill. In the same year Sovietskoye Radio published a Russian translation of the book.

He is also interested in pattern recognition and has written a number of papers in that field. In 1968 he published the book "Recognition of Patterns Using the Frequencies of Occurrence of Binary Words". A third, revised and enlarged edition appeared in 1978, published by Springer-Verlag. In 1972 he published the book "An Introduction to the Design of Pattern Recognition Devices" with Springer-Verlag.

Dr. Becker is Chairman of the Danish Pattern Recognition Society, a society he founded in 1976. He is the Chairman of the Nominating Committee, International Association for Pattern Recognition. Also, he is Chairman of the Denmark Section of the Institute of Electrical and Electronics Engineers.

# sets and systems

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## Aims and Scope:

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Fuzzy Sets have become a concern and a tool of persons working in very different areas: mathematicians in many different areas such as logic, topology, algebra, etc. are doing research in the area of Fuzzy Sets. The same is being done in the area of general system (intelligent systems) of decision theory and optimization, of approximate reasoning and sociology. Moreover, the theory of Fuzzy Sets is already applied in a variety of areas such as control theory, medicine, information processing, cluster analysis, genetics, electrical engineering etc..

Most of the contributions to the theory of Fuzzy Sets and of their applications are distributed over a very wide range of scientific journals, thus the transparency of knowledge in the area of Fuzzy Sets nowadays is very small, and scientists working in the area of fuzzy sets in different disciplines or on different continents do not know of each other and of their contributions.

## Recent Papers:

Decision-making with a fuzzy preference relation *(S. A. Orlovsky).* Decision-making in a fuzzy environment with fuzzy termination time *(J. Kacprzyk).* Fuzzy games: A description of the concept *(D. Butnariu).* Fuzzy subobjects in a category and the theory of $\mathcal{C}$-sets *(D. Ralescu).* The algebra of fuzzy logic *(P. Albert).* Fuzzy mappings and algebraic structures *(J. Nieminen).*

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# *Letter to the Editor*

Sir—I am grateful to Professor Suppe for his excellent review[1] of my "Theory of Modelling and Simulation". He raises a couple of points, which, in the spirit of his astute discussion, I should like to comment upon.

1) Suppe asserts that my theory is not complete although comprehensive. To this I agree without qualification. He points to my formulation of probabilistic systems as deterministic systems with pseudorandom number generators as a theoretical, if not practical limitation; and he asserts that real system structures and morphisms are unlikely to be of this form.

To me the case is not so clear cut. In the first place, structure is a property of *models* not of reality. The choice of a structural formalism to represent a given real system depends on factors such as expressive power and computational efficiency as well as adequacy. Different structures may be employed to represent the same system for different purposes. The usual stochastic process formalism may be convenient analytically, while the deterministic formulation may be more appropriate in a simulation context. The crucial point, which the theory begins to address, is that there be means of intertranslating from one formalism to another so that an integrated approach to modelling is possible.

2) Suppe asserts that my formulation of model validation "requires evidence which virtually precludes every logically possible alternative model". Since "obtaining such evidence is virtually impossible" (I would omit the "virtually"), I am led to the "skeptical" position of considering validation only within experimental frames. Actually, I believe the experimental frame concept is fundamental to decomposing reality into manageable pieces independently of the validation problem. Moreover, complete validation is theoretically impossible within a frame also. The point is that when a model is considered in isolation, a finite amount of evidence can only justify partial confidence in its validity.

My interpretation of what Suppe is saying is that a model should not be considered in isolation. This seems to be the brunt of his discussion of "reasoning patterns which typically involve only considering those hypotheses which are highly plausible in the light of current knowledge". It is fair to say that we do not well understand the linkages by which empirical data, hypotheses, models, and theories are interconnected to constitute "current knowledge". But it should be obvious that while a model may acquire a degree of confidence well beyond that justified by its empirical validation in isolation, this extra confidence is conditional on the validity of "current knowledge" viewed as a single compound theory. But the latter is open to question since in the process of successive plausibility-based validations, the theoretical content of "current knowledge" may outstrip its empirical support. Thus extreme "antiskeptism" may in the end be as self-defeating as extreme "skeptism".

What is needed is an improved understanding of the structure of scientific knowledge and of the strategies by which knowledge is acquired and integrated into the existing whole. In the book under review I indicated that the eye-tooth-cutting test of the theory would be its application to integrated modelling. Subsequent work[2,3,4] has gone some way toward taking into account the integrative factors, stressed by Suppe, in the above modelling context. I am thankful to him for bringing them to the fore.

NOTES

1. "Review of *Theory of Modelling and Simulation*", by Frederick Suppe, I.J.G.S., **4**, No. 2, 1978, pp. 127–131.
2. "Structuring the Organization of Partial Models" puts forth the basic approach and appeared in the same issue of I.J.G.S. as Suppe's review (**4**, No. 2, 1978, pp. 81–88).
3. B. Zeigler, "Multi-level, Multi-formalism Modelling: An Ecosystem Example". In: *Theoretical Systems Ecology*, edited by E. Halfon, Academic Press, 1979.
4. Several papers in *Methodology in Systems Modelling and Simulation*, edited by B. P. Zeigler et al., North Holland (1979) deal with this topic.

BERNARD P. ZEIGLER
*Weizmann Institute of Science,*
*Rehovot, Israel*

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# NOTES FOR CONTRIBUTORS

## TYPESCRIPTS

Papers should be typed on good quality paper with double spacing and wide (3 cm) margins, using one side only, and submitted in triplicate to the editor: **George J. Klir, Department of Systems Science, School of Advanced Technology, State University of New York, Binghamton, New York 13901, U.S.A.**

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1. L. A. Zadeh, "Similarity relations and fuzzy orderings." *Information Sciences*, **3**, No. 2, April 1971, pp. 171-200.

*Books*: Author's initials and last name, title of the book (underlined and followed by a period), publisher, location, year, chapter or page numbers if desired.

2. G. J. Klir, *An Approach to General Systems Theory* Van Nostrand Reinhold, New York, 1969.

*Edited collections*: Author's initials and last name, title of the paper enclosed in quotation marks (followed by a period), name of the volume (underlined), editor's name, publisher, location, year, inclusive pages.

3. W. R. Ashby, "Systems and their informational measures." In: *Trends in General Systems Theory*, edited by G. J. Klir, John Wiley, New York, 1972, pp. 78-97.

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